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First problem

Let H be a Hilbert space, $\varphi \in H^*$ and $T \in \mathcal{L}(H, H)$. Let us consider the functional $\psi := \varphi \circ T$. Because ψ is a composition of two linear maps, it is a linear map. Also we have $\|\psi\| = \|\varphi \circ T\| \leq \|\varphi\| \cdot \|T\|$ and because φ and T are bounded, ψ is bounded. From the Riesz representation theorem we obtain that there exists uniquely determined $y \in H$ such that for all $x \in H$ we have $\psi(x) = \langle x, y \rangle$ (i.e. $\varphi(Tx) = \langle x, y \rangle$).