

Geometric Group Theory Student Workshop 2013 - Abstracts

When the random group collapses

Sylwia Antoniuk (Adam Mickiewicz University)

Let $\Gamma(n, p)$ be a group generated by a random group presentation $\langle S | R \rangle$ with n generators, in which relators from R are cyclically reduced words of length three and each such word is present in R independently with probability p . The above model is in some manner equivalent to Żuk's model of a random triangular group. It is known that for $\varepsilon > 0$ and $p > n^{-3/2+\varepsilon}$ asymptotically almost surely the random group $\Gamma(n, p)$ is trivial, whereas for $p < n^{-3/2-\varepsilon}$ this group is infinite and hyperbolic. We show that the first of the above properties can be strengthened, namely we show that there exists a constant $C > 0$ such that if $p > Cn^{-3/2}$ then asymptotically almost surely $\Gamma(n, p)$ is trivial.

Computability and decision problems in group theory

Maurice Chiodo (University of Milan)

Computability plays an important part in many branches of mathematics, especially in the study of groups. With a well-defined concept of "algorithm", we can not only describe problems which are computable, but also prove that certain problems are incomputable. In this talk I will give a description of the notion of an "algorithm" by defining a Turing machine, and explain how this can be used to show that many problems in group theory (as well as problems in geometry and topology) are provably incomputable.

Realizations of unitary representations

Lukasz Garncarek (University of Wrocław)

A realization of a unitary representation is an equivalent representation, in which the underlying Hilbert space belongs to some category of "concrete" Hilbert spaces (e.g. the L^2 spaces). I will speak about some ideas related to this notion. There will be many questions

and no answers at all.

L^2 invariants and decidability

Lukasz Grabowski (University of Oxford)

I will start by introducing L^2 invariants of groups - these are invariants defined by looking at the boundary map in CW-complexes, treated as an operator on an infinite-dimensional Hilbert space. An example is the random walk operator on \mathbb{Z}^2 - we will see how this and other operators arise as boundary maps of CW-complexes. The main part of the lecture will focus on cases where concrete calculations are possible, i.e. when we have a group which is a semidirect product, acting on a manifold. As the main application, we will see how one can study classical problems from group ring theory - in particular, the problem of the existence of zero divisors - from the perspective of decidability theory.

Small cancellation groups and Greendlinger lemma.

Katarzyna Jankiewicz (University of Warsaw)

I will introduce the classical small cancellation conditions, i.e. requirement that relations in presentation of a group have small overlaps with each other. I will give a proof of the Greendlinger lemma, the fundamental theorem in the theory of small cancellation groups, and discuss some applications of it.

Random groups and Property (T)

Marcin Kotowski, Michał Kotowski (University of Toronto)

In our lectures we will introduce the Gromov model of random groups, the notion of Kazhdan's Property (T) and prove that random groups for densities $d > 1/3$ satisfy Kazhdan's Property (T). Along the way we will prove the spectral criterion for property (T) and review relevant results from spectral graph theory and random graphs (random regular graphs and Erdős-Rényi graphs).

Uniformly finite homology

Michał Marcinkowski (University of Wrocław)

This is a coarse version of homology theory defined by Block and Weinberger. Its relevance to geometric group theory is justified by the fact that vanishing of the zero homology characterises non-amenability. In this talk I will provide some other applications of uniformly finite homology; among others (if time permits) recent homological characterisation of essential manifolds with maximal macroscopic dimension (after A.Dranishnikov).

The isoperimetric inequality and its applications

Tomasz Odrzygóźdź (University of Warsaw)

I will introduce the notion of Gromov's Random Group at density d and prove the Isoperimetric inequality for planar diagrams associated with this group (van Kampen diagrams). Then I will show applications of this inequality: determining the topology of the presentation complex, proving that group is hyperbolic for some densities. Moreover I'm going to show some results, which I have obtained during writing my thesis - I will present square random group model and talk about it's connections with Gromov's model and the triangular model.

Automorphisms of surfaces

Piotr Przytycki (IMPAN)

In this lecture course we will give a proof of Thurston's classification of surface automorphisms. The theorem says that every surface automorphism is periodic, reducible or pseudo-Anosov. We will follow the book of Casson and Bleiler, where (measured) geodesic laminations are used.

Helly's Theorem for systolic complexes

Krzysztof Świącicki (University of Warsaw)

My short lecture will be divided in to three parts. First I will recall basic properties of systolic complexes, which are symplcial analogues of nonpositively curved spaces and inherit lots CAT(0)-like properties. Then I will state Helly's Theorem for Euclidean spaces and CAT(0) cube complexes. After that introduction I will prove version of Helly's Theorem for 7-systolic complexes.

TBA

Robert Tang (University of Warwick/IMPAN)

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