Program and Book of Abstracts

Workshop
Crossing the borderlines in fluids and biology

Chęciny, 12 – 16.06.2023

Organizers:

Miroslav Bulíček,
Jakub Skrzeczkowski,
Agnieszka Świerczewska-Gwiazda

Program of the Workshop

Monday, 12.06.2023

17:00	**Departure of the bus from Warsaw to Chęciny**
20:00	**Dinner**

Tuesday, 13.06.2023

9:00	**Breakfast**
9:55 - 10:00	**Opening of the workshop**
10:00 - 10:40	**Cleopatra Christoforou** *An Euler-type flocking model: Weak solutions with bounded support and long-time behavior*
10:45 - 11:25	**Piotr Gwiazda** *From compressible Euler system to porous medium equation*
11:30 - 12:00	**Coffee break**
12:00 - 12:25	**Dimitrios Katsaounis** *Hybrid modelling for cancer invasion and metastasis*
12:30	**Lunch**
13:30 - 19:00	**Kayaking trip**
19:00	**Dinner**

Wednesday, 14.06.2023

9:00	**Breakfast**
10:00 - 10:40	**Noemi David** *Singular limits arising in mechanical models of tumor growth*
10:45 - 11:25	**Tomasz Dębiec** *On some Hookean models of dilute polymeric fluids*
11:30 - 12:00	**Coffee break**
12:00 - 12:40	**Petr Kaplický** *Stability of equilibria to generalized Navier-Stokes-Fourier system*
12:45 - 13:10	**Alejandro Ramos Lora** *Study of the asymptotic behavior for the NNLIF neuronal models*
13:30	**Lunch**

15:00 - 15:40	**Miroslav Bulíček** *Existence analysis of incompressible and compressible fluid models*
15:45 - 16:10	**Stanislav Mosný** *Data assimilation in the theory of non-Newtonian fluids*
16:15 - 16:45	**Coffee break**
16:45 - 17:25	**Jakub Woźnicki** *Non-Newtonian fluids with discontinuous-in-time stress tensor*
19:00	**Dinner**

Thursday, 15.06.2023

9:00	**Breakfast**
10:00 - 10:40	**Lara Trussardi** *Nonlocal-to-local convergence of Cahn-Hilliard equations*
10:45 - 11:25	**Jan Peszek** *Singular alignment dynamics*
11:30 - 12:00	**Coffee break**
12:00 - 12:40	**Charles Elbar** *A compressible Cahn-Hilliard-Navier-Stokes for the modelling of tumor growth*
13:30	**Lunch**
15:00 - 15:40	**Jakub Skrzeczkowski** *Nonlocal effects and degenerate Cahn-Hilliard equation*
15:45 - 16:10	**Ondřej Váša** *Differential equations with eigenvalue in boundary conditions*
16:15 - 16:45	**Coffee break**
16:45 - 17:25	**Agnieszka Świerczewska-Gwiazda** *Onsager's Conjecture for General Conservation Laws*
19:00	**Dinner and bonfire**

Friday, 16.06.2023

8:00	**Breakfast**
9:00	**Departure from Chęciny to Warsaw**

Existence analysis of incompressible and compressible fluid models for heat-conducting and chemically reacting mixtures

Miroslav Bulíček

Charles University, Prague

Abstract

We consider a flow of heat conducting chemically reacting mixtures in dimension two and three. We focus on both compressible and incompressible setting whose thermodynamics is described by general free energies satisfying some fundamental structural assumptions. We discuss the conditions leading to the existence of weak solution. It is noticeable that the considered models are thermodynamically consistent on one hand and are able to cover the classical models like the Maxwell-Stefan cross-diffusion equations in the Fick-Onsager form as a special case on the other hand. Compared to previous works, a very general model class is analysed, including cross-diffusion effects, temperature gradients, compressible fluids, and different molar masses (in case of compressible fluid). In addition, the technique leading to the compactness of the pressure is a nontrivial generalisation of the use Feireisl's oscillation defect measure and the newly developed technique relies heavily on the convexity of the free energy and the strong convergence of the relative chemical potentials.

An Euler-type flocking model: Weak solutions with bounded support and long-time behavior

Cleopatra Christoforou

University of Cyprus

Abstract

In this talk, we present some results on the existence and time-asymptotic flocking of weak solutions to a hydrodynamic model of flocking-type with an all-to-all interaction kernel in a one-space dimension. An appropriate notion of entropy weak solutions with bounded support is given to capture the behavior of solutions with initial data that has finite total mass confined in a bounded interval and initial density uniformly positive therein. We show global in time existence of entropy weak solutions with concentration for any initial data of bounded variation having the structure above. In addition, we will present a recent result on the time-asymptotic limit for such solutions, showing the asymptotic decay towards flocking profiles without any further restrictions on the data. Joint work with Debora Amadori (Univ. of L'Aquila).

Singular limits arising in mechanical models of tumor growth

Noemi David

Institute Camille Jordan, Université de Lyon

Abstract

The mathematical modelling of cancer has been increasingly applying fluid-dynamics concepts to describe the mechanical properties of tissue growth. The bio-mechanical pressure plays a central role in these models, both as the driving force of cell movement and as an inhibitor of cell proliferation. In this talk, I will present how it is possible to build a bridge between models that have different pressure-velocity or pressure-density relations. In particular, I will focus on the inviscid limit from a visco-elastic model to a Darcy's law-based model, and the incompressible limit that links the latter to a Hele-Shaw free boundary problem with density constraint.

On some Hookean models of dilute polymeric fluids

Tomasz Dębiec

University of Warsaw

Abstract

We consider the Hookean dumbbell model, a system of nonlinear PDEs arising in the kinetic theory of homogeneous dilute polymeric fluids. It consists of the unsteady incompressible Navier-Stokes equations in a bounded Lipschitz domain, coupled to a Fokker-Planck-type parabolic equation with a centre-of-mass diffusion term, for the probability density function, modelling the evolution of the configuration of noninteracting polymer molecules in the solvent. The micro-macro interaction is reflected by the presence of a drag term in the Fokker-Planck equation and the divergence of a polymeric extra-stress tensor in the Navier-Stokes balance of momentum equation. In a simplified case where the drag term is corotational, we prove global existence of weak solutions and discuss some of their properties: we use the relative energy method to deduce a weak-strong uniqueness type result, and derive the macroscopic closure of the kinetic model: a corotational Oldroyd-B model with stress-diffusion. In the general noncorotational case we consider "generalised dissipative solutions" — a relaxation of the usual notion of weak solution, allowing for the presence of a, possibly nonzero, defect measure in the momentum equation, which accounts for the lack of compactness in the polymeric extra-stress tensor. Joint work with Endre Suli (Oxford).

A compressible Cahn-Hilliard-Navier-Stokes for the modelling of tumor growth

Charles Elbar

Laboratoire Jacques-Louis Lions, Sorbonne Université

Abstract

Motivated by the mathematical modeling of tumor invasion in healthy tissues, we propose a generalized compressible diphasic Cahn-Hilliard-Navier-Stokes model. We assume that the fluid's two phases represent two different cell populations: cancer cells and healthy tissue and we include in the model possible friction and proliferation effects. The model aims to be as general as possible to study the possible mechanical effects playing a role in the invasive growth of a tumor. The aim of this talk is to discuss the analytical and numerical challenges behind this model.

From compressible Euler system to porous medium equation

Piotr Gwiazda

Institute of Mathematics, Polish Academy of Sciences

Abstract

This talk will be devoted to high friction and long time limit from compressible Euler to some target systems. In the classical case of isentropic Euler the example of target system is the porous medium equation. The limit will be done by relative entropy method, in the class of measure-valued solutions.

Stability of equilibria to generalized Navier-Stokes-Fourier system

Petr Kaplický

Charles University, Prague

Abstract

We consider a generalized Newtonian incompressible heat conducting fluid with prescribed nonuniform temperature on the boundary and with the no-slip boundary conditions for the velocity. The fluid occupies a three dimensional domain. No external body forces are applied to the fluid. We identify different classes of proper solutions in dependence on the growth of the constitutively determined part of the Cauchy stress that converge to the equilibria exponentially fast in a suitable metric. Consequently, the equilibrium is nonlinearly stable and attracts all weak solutions from these classes. We also show that these classes are nonempty.

Hybrid modelling for cancer invasion and metastasis

Dimitrios Katsaounis

University of St Andrews

Abstract

Cancer cells have the ability to interact with the tumour microenvironment and invade the surrounding tissue by reformulating the extracellular matrix (ECM). The coordinated actions of cancer cells, the ECM, cancer associated fibroblasts (CAFs), and the epithelial to mesenchymal transition (EMT) result to in the invasion of the tissue. In this talk, I will present a multiscale hybrid mathematical model which combines the macroscopic nature of the phenomenon, where solid tumours of epithelial-like cancer cells (ECCs) invade the tissue, as well as the microscopic individual based strategy of mesenchymal-like cancer cells (MCCs). The model consists of partial and stochastic differential equations that describe the evolution of the ECCs and the MCCs while accounting for the transitions between them. Numerical simulations of the proposed model will be presented.

Data assimilation in the theory of non-Newtonian fluids

Stanislav Mosný

Charles University, Prague

Abstract

The Ladyzhenskaya model is a generalization of the Navier-Stokes model that can be used to describe the behavior of non-Newtonian fluids. In our talk, we will first discuss the well-posedness of this problem. Our main goal is to study data assimilation for the Ladyzhenskaya model, which aims to find an approximation of the reference solution when the initial data is not known. Instead, we have measurements of the reference solution. We will show that the data assimilation algorithm constructs an approximation that converges to the reference solution over time. The key aspect of this study is the analysis of the long-term behavior of the reference solution.

Singular alignment dynamics

Jan Peszek

University of Warsaw

Abstract

Classical micro-meso-macroscopic analysis of hard-spheres dynamics (leading from Newtonian systems to Eulerian dynamics) is a staple point in mathematical physics. Similar program can be performed for the alignment dynamics, the main difference being, that the considered systems are purely non-local at every scale. In particular, singular alignment dynamics leads from systems of ODEs to the so called fractional Euler-alignment system, with viscosity encoded by a fractional Laplacian weighted by the solution itself. Remarkably, the rigorous passage from micro- to meso- and macroscopic scale can be approached using a deterministic mean-field limit in Dobrushin style.

I will present the latest results and ideas related to the micro- to meso- and macroscopic limit for singular alignment dynamics. This includes the heterogeneous gradient flows related to singular alignment (joint with David Poyato, University of Granada) with matrix valued communication and a monokineticity estimate for strongly singular alignment (joint with Michał Fabisiak, University of Warsaw).

Study of the asymptotic behavior for the NNLIF neuronal models

Alejandro Ramos Lora

University of Granada

Abstract

In recent years, there has been a significant body of research focused on investigating neural networks through the lens of Partial Differential Equations (PDEs). Within this domain, Nonlinear Noisy Leaky Integrate and Fire neuronal (NNLIF) models have emerged as a important area of study [1]. Various studies have explored the existence of solutions and long-term behavior of these systems [2, 3, 4, 5]. Building upon existing literature and incorporating our own analytical and numerical investigations (see out previous work [6]), we have delved deeper into understanding the asymptotic properties of the solutions to these equations.

In this work we first perform a study of the asymptotic properties of the linear model, as well as its regularization and existence properties, in a norm that we find convenient for the subsequent analysis of the nonlinear equation. We then propose a perturbative argument that allows us to expand our results to the nonlinear model, with certain restrictions on the initial condition, or the relative weight of the nonlinear term. The outcome of our research enables us to advance upon previously published results concerning the stability of solutions to these equations.

1. N. Brunel, *Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons.* Journal of Computational Neuroscience 8, 3 (2000), 183–208.

2. M. J. Cáceres, J. A. Carrillo, B. Perthame. *Analysis of nonlinear noisy integrate & fire neuron models: blow-up and steady states.* The Journal of Mathematical Neuroscience 1.1 (2011) 1-33.

3. J A. Carrillo, M. d. M. González, M. P. Gualdani, M. E. Schonbek. *Classical Solutions for a Nonlinear Fokker-Planck Equation Arising in Computational Neuroscience.* Communications in Partial Differential Equations. 38:3 (2013) 385-409.

4. J. A. Carrillo, B. Perthame, D. Salort, D. Smets. *Qualitative properties of solutions for the noisy integrate and fire model in computational neuroscience.* Nonlinearity 28.9 (2015) pp.3365.

5. M. J. Cáceres, P. Roux, D. Salort, R. Schneider. *Global-in-time solutions and qualitative properties for the NNLIF neuron model with synaptic delay.* Communications in Partial Differential Equation. 44 (12) (2019) 1358-1386.

6. M. J. Cáceres, R-L. An *Understanding of the Physical Solutions and the Blow-up Phenomenon for Nonlinear Noisy Leaky Integrate and Fire Neuronal Models.* Communications in Computational Physics. 30 (3) (2021) 820-850.

Non-local effects and degenerate Cahn-Hilliard equation

Jakub Skrzeczkowski

University of Warsaw

Abstract

I will discuss several situations when one has to perform limit passage from non-local to local operators in the context of the degenerate Cahn-Hilliard equation (i.e. with degenerate mobility). This includes kinetic derivation of the equation (*Comm. Math. Phys.*, 2023, with C. Elbar, M. Mason, B. Perthame), fairly classical problem of passage to the limit from non-local to local equation (*JDE*, 2023, with C. Elbar) and the same problem for aggregation-diffusion system (arXiv:2303.11929, together with J. A. Carrillo, C. Elbar). Not all of these problems are fully understood and to some of them, solutions are available only on the torus.

Onsager's Conjecture for General Conservation Laws

Agnieszka Świerczewska-Gwiazda

University of Warsaw

Abstract

A common feature of systems of conservation laws of continuum physics is that they are endowed with natural companion laws which are in such case most often related to the second law of thermodynamics. This observation easily generalizes to any symmetrizable system of conservation laws. They are endowed with nontrivial companion conservation laws, which are immediately satisfied by classical solutions. Not surprisingly, weak solutions may fail to satisfy companion laws, which are then often relaxed from equality to inequality and overtake a role of a physical admissibility condition for weak solutions. We want to discuss what is a critical regularity of weak solutions to a general system of conservation laws to satisfy an associated companion law as an equality. An archetypal example of such result was derived for the incompressible Euler system by Constantin et al. in the context of the seminal Onsager's conjecture. This general result can serve as a simple criterion to numerous systems of mathematical physics to prescribe the regularity of solutions needed for an appropriate companion law to be satisfied.

Nonlocal-to-local convergence of Cahn-Hilliard equations

Lara Trussardi

University of Konstanz

Abstract

The Cahn-Hilliard equation is widely used in the study of phase field models. A nonlocal version of the equation, proposed by Giacomin and Lebowitz, attracted great interest in recent years. In this talk we show existence and uniqueness of solutions for nonlocal Cahn-Hilliard equations with degenerate potential and present the convergence of a nonlocal version of the Cahn-Hilliard equation to its local counterpart as the nonlocal convolution kernel approximates a Dirac delta in a periodic and Neumann boundary conditions setting. This is based on a series of joint works with E. Davoli, H. Ranetbauer, and L. Scarpa.

Differential equations with eigenvalue in boundary conditions

Ondřej Váša

Charles University, Prague

Abstract

The goal of this work was to study Stokes problem with eigenvalue in bound- ary condition. We were in particular interested in determining the asymptotic behaviour of the sequence of eigenvalues. We wanted to modify techniques used in papers studying asymptotics of eigenvalues in boundary condition for Steklov problem and conclude similar results. Firstly, we proved that there are only countably many eigenvalues for our problem. Next, we explicitly calculated asymptotics of eigenvalues of auxiliary problems on simple domains. Finally, by using min-max theorem we managed to get desired estimates of eigenvalues of the original problem on general domains by previous results and hence obtain the desired results.

Non-Newtonian fluids with discontinuous-in-time stress tensor

Jakub Woźnicki

Institute of Mathematics, Polish Academy of Sciences

Abstract

We consider the system of equations describing the flow of incompressible fluids in bounded domain. In the considered setting, the Cauchy stress tensor is a monotone mapping and has asymptotically $(s-1)$-growth with the parameter s depending on the spatial and time variable. We do not assume any smoothness of s with respect to time variable and assume the log-Hölder continuity with respect to spatial variable. Such a setting is a natural choice if the material properties are instantaneous, e.g. changed by the switched electric field. We establish the long time and the large data existence of weak solution provided that $s \geq \frac{3d+2}{d+2}$.