

## Miniproject 1:

### Schauder's estimates for Laplacian.

The target is to prove that if  $f \in C^{0,d}(B_r)$  and  $-\Delta u = f$  then  $u \in C^{2,d}(B_r)$ . Moreover, it is not true for  $d=0$ . This represents some gain of regularity (two derivatives) for elliptic equations.

Our reference: [RO] Regularity Theory for Elliptic PDEs  
Ros-Oton, Fernandez Real.  
[E] Partial Differential Equations,  
Evans

### Talks:

- ① Introduce Hölder spaces  $C^{0,d}$ ,  $C^{1,d}$ ,  $C^{2,d}$ , page 6. Then prove property (H8) on compactness and interpolation inequality (1.13). [RO]
- ② Discuss covering lemma (Remark 2.15). [RO]
- ③ Prove that  $u(x,y) = \log \log(x^2+y^2) (x^2-y^2)$  is not  $C^2$  but  $\Delta u$  is continuous, see discussion on page 34. [RO]

- (4) Prove estimate for harmonic functions as in Theorem 7 [E], page 29.
- (5) State and prove Theorem 2.14 — present the first proof (there are two in the book). [RO]
- (6) State and prove Corollary 2.16. [RO]

### Comments:

- (A) Topics 1–3 are easy.
- (B) Topic 5 is for two students with strong background in analysis.
- (C) Topic 6 requires understanding of mollification