# Series 13

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**Exercises:**

(A1) Show that the norms $\| \cdot \|_1, \| \cdot \|_2, \| \cdot \|_\infty$ are equivalent on $\mathbb{R}^n$. In particular, show that for any $x \in \mathbb{R}^n$

$$\|x\|_\infty \leq \|x\|_2 \leq \|x\|_1$$

and

$$\|x\|_1 \leq \sqrt{n}\|x\|_2 \leq n\|x\|_\infty.$$

(A2) Calculate the limits or show that the limit does not exist.

- $$\lim_{(x,y)\to(0,0)} \frac{x^3 + y^4}{x^2 + y^4}$$
  ,
- $$\lim_{(x,y)\to(0,0)} \frac{x^5 + y^4}{x^4 + y^2}$$
  .

(A3) Determine whether the function $f : \mathbb{R}^3 \to \mathbb{R}^2$

$$f(x, y, z) = \begin{cases} \left(\frac{xy}{1+z^2}, \frac{x^2y^2z^2}{x^2+y^2+z^2}\right), \text{ for } (x, y, z) \neq (0, 0, 0), \\ 0, \text{ otherwise,} \end{cases}$$

is continuous.

(A4) Determine the existence of limits of a function $f(x, y) = \frac{y \sin(\pi x)}{x+y-1}$ at the points lying on the line $x + y - 1 = 0$.

(A5) Calculate partial derivatives of functions

- $f(x, y) = |xy|$,
- $f(x, y) = \frac{\sin(x)}{y}$,
- $f(x, y, z) = e^x \sin(y) + e^y \sin(2z) + e^z \sin(3x)$.

(A6) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x, y) = \begin{cases} \frac{yx^2}{x^2+y^4}, \text{ for } (x, y) \neq (0, 0), \\ 0, \text{ otherwise,} \end{cases}$$

Calculate

$$\frac{\partial}{\partial x} f(x, y), \quad \frac{\partial}{\partial y} f(x, y).$$

Is $f$ continuous at 0? Does the existence of partial derivatives implies the continuity of the function? Compare with the one dimensional case.