Series 14

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Exercises:

(A1) Calculate the gradient of ‘

$$f(x) = e^{\cos(x^2 - y^2)} \ln\left(1 + \sin\left(\operatorname{tg}\left(\frac{x+y}{2}\right)\right)\right)$$

at $(0,0)$.

(A2) Calculate the derivative of $F : \mathbb{R}_+ \times (-\pi, \pi) \times \mathbb{R} \to \mathbb{R}^3$

$$F(r, \theta, z) = (r\cos(\theta), r\sin(\theta), z).$$

(A3) Determine whether the function

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2}, \text{ when } (x,y) \neq (0,0) \\ 0, \text{ when } (x,y) = (0,0). \end{cases}$$

is differentiable at $(0,0)$.

(A4) Depending on the parameter $p > 0$ determine whether the function

$$f(x,y) = \ln(1 + |xy|^p)$$

is differentiable at $(0,0)$.

(A5) Find the derivative of F^{-1}, when

$$F(x,y) = (e^x + e^{3y}, e^{3y}).$$

(A6) Find the derivative of $F : \mathbb{R}^3 \to \mathbb{R}^2$ at $(0,0,0)$, when

$$F(x,y,z) = g(e^y + \cos(x), e^{\sin(z)}, e^{x+y}\sin(z)),$$

knowing that the derivative of $g : \mathbb{R}^3 \to \mathbb{R}^2$ at $(2,1,0)$ is given by

$$D_{(2,1,0)}g = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \end{pmatrix}.$$