Series 15

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Exercises:

(A1) Show that the function $f(x,y) = (x-y^2)(3x-y^2)$ restricted to any line passing through the point $(0,0)$ has a local minimum at $(0,0)$. Does the function f have a local minimum at $(0,0)$?

(A2) Find the maximum and the minimum of the function

$$f(x,y) = x(y-x)e^{-y}$$

on the set $A = \{(x,y) \in \mathbb{R}^2 : -1 \leq 3x \leq 2y \leq 6\}$.

(A3) Find the maximum and the minimum of the function

$$f(x,y) = \frac{y}{x^2+4y^2+1}$$

on the set $A = \{(x,y) \in \mathbb{R}^2 : 0 \leq x\}$.

(A4) Find the maximum and the minimum of the function

$$f(x,y,z) = 6xy - 3xz - 2yz$$

on the set $A = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq x,y,z \leq 1\}$.

(A5) Find a point lying on the surface $\{(x,y,z) \in \mathbb{R}^3 : z = xy+1\}$, which lies closest to the origin of the coordinate system.

Bonus exercise:

(Z1) Suppose, that for a function $F : \mathbb{R}^2 \to \mathbb{R}$ there exists a limit $\lim_{(x,y)\to(0,0)} F(x,y)$. Show, that if

$$\lim_{x\to 0}\lim_{y\to 0} F(x,y), \quad \lim_{y\to 0}\lim_{x\to 0} F(x,y),$$

exist, then

$$\lim_{x\to 0}\lim_{y\to 0} F(x,y) = \lim_{y\to 0}\lim_{x\to 0} F(x,y) = \lim_{(x,y)\to(0,0)} F(x,y).$$