Series 16

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Exercises:

(A1) Find the supremum of the function

$$f(x,y) = \frac{(x^2+y^2-x)(1-x)^2}{(x-y)^2}$$

on the set $A = \{(x,y) \in \mathbb{R}^2 : \, x^2 - 2x + y^2 \geq 0, 0 \leq x < 1, 0 < y \leq 1\}$.

(A2) Find the supremum and the infimum of the function

$$f(x,y) = \frac{x^2 y}{x+1} e^{-xy},$$

on the set $A = \{(x,y) \in \mathbb{R}^2 : \, 0 \leq y \leq x\}$.

(A3) Find the supremum and the infimum of the function

$$f(x,y) = \ln(1 + x^2 + y^2),$$

on the set $A = \{(x,y) \in \mathbb{R}^2 : \, x^2 + y^2 \leq 1, x \leq y\}$.

(A4) Find the supremum and the infimum of the function

$$f(x,y) = \frac{y-x}{x^4+1},$$

on the set $A = \{(x,y) \in \mathbb{R}^2 : \, 0 \leq y \leq x\}$.

(A5) Find the points in which the gradient of the following function is 0. Discuss in which of those the given function has a local minimum, local maximum or neither.

(a) $f(x,y,z) = x^2 + y^2 + z^2 + 2x + 4y - 6z$,

(b) $f(x,y) = x^3 + 3xy^2 - 15x - 12y$,

(c) $f(x,y,z) = xy^2z^3(6 - x - 2y - 3z)$,

(d) $f(x,y,z) = x + \frac{4y^2}{x} + \frac{z^2}{y} + \frac{2}{z}$,

(e) $f(x,y,z) = \frac{2x^2}{y} + \frac{y^2}{z} + 8z^2 - 8x$,

(f) $f(x,y) = -x^4 + y^4 + 4x^2y - 2y^2$.

(A6) Show that

$$F(x,y) = (1 + e^y)\cos(x) - ye^y$$

has infinitely many local maxima, but no local minima.

(A7) Function $f : \mathbb{R}^2 \to \mathbb{R}$ is twice differentiable and satisfies at any point

$$f_{xx} + f_{yy} > 0.$$

Suppose that the given function has only non-degenerate critical points (i.e. its hessian matrix is invertible). Show that f has no local maxima.

(A8) Determine for which $a \in \mathbb{R}$ the function $f(x,y) = \cos(x+y) + a\mathrm{tg}(xy)$ has at $(0,0)$ local minimum, local maximum or neither.