Series 17

Jakub Woźnicki

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Reminder: Suppose $g \in C^1(\Omega; \mathbb{R})$ and $F = (F_1, ..., F_m) \in C^1(\Omega; \mathbb{R}^m)$, where $\Omega \subseteq \mathbb{R}^n \times \mathbb{R}^m$ is an open subset. Let $M = \{z \in \Omega : F(z) = 0\}$ and fix $p \in M$ for which $DF(p)$ is a linear epimorphism. Then, the necessary condition for g to achieve a local minimum or maximmum in p, on the set M, is that

$$\nabla g(p) \cdot w = 0 \text{ for any } w \in T_p(M) \text{ (recall that } T_p(M) = \ker DF(p))$$

and there exist $\lambda_1, ..., \lambda_m$ such that

$$\nabla g(p) = \sum_{i=1}^{m} \lambda_i \nabla F_i(p).$$

Exercises:

(A1) Calculate the Taylor series of order 3 at point $a = (1, 1)$ of the function

$$f(x, y) = \frac{x}{y} - \frac{y}{x}.$$

(A2) Function $f \in C^2(\mathbb{R}^2)$ satisfies

$$\lim_{(x,y)\to(0,0)} \frac{f(x, y) - \operatorname{tg}(x)\sin(y)}{x^2 + y^2} = 0.$$

Find $f_{xy}(0, 0)$.

(A3) Function $f \in C^2(\mathbb{R}^2)$ satisfies

$$\lim_{(x,y)\to(0,0)} \frac{f(x, y) - \ln(1 + 3x)\cos(y)}{x^2 + y^2} = 1.$$

Find $f_{xx}(0, 0)$.

(A4) Find the supremum of the function $f(x, y, z) = xyz$ on the set

$$A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 9,\ xy + yz + zx = 8\}.$$

(A5) Find the surpemum of the function $f(x, y, z) = x + z$ on the set

$$A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1,\ x^2 - x + y^2 = 1,\ z \geq 0\}.$$