Series 19

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29.05.2023

Exercises:

(A1) Calculate the measure of

- a set $\mathcal{C} \subset [0,1]$ of all of the numbers whose expantion in base 3 does not contain a 1 (or contains only one 1 and it is the last non-zero number in the expansion) (Cantor's set),
- a set $\mathcal{D} \subset [0,1]$ of all of the numbers whose decimal expansion does not contain a 1.

(A2) Define a function $F : [0,1) \to [0,1)$ by $F(x) = 2\,x - \lfloor 2\,x \rfloor$. Suppose $A \subset [0,1)$ is a measurable set. Show that $F^{-1}(A)$ is measurable and calculate its measure.

(A3) Calculate the following limits

a)
$$\lim_{n\to+\infty} \int_0^n \left(1 + \frac{x}{n}\right)^{n+1} e^{-2x}\,\mathrm{d}x,$$

b)
$$\lim_{n\to+\infty} \int_{e^{-n}}^1 \left(1 + \frac{\ln(x)}{n}\right)^{n} x^{-\frac{3}{2}}\,\mathrm{d}x,$$

c)
$$\lim_{n\to+\infty} \int_0^1 n\,e^{-nx}\,\mathrm{d}x,$$

d)
$$\lim_{n\to+\infty} \int_0^1 \frac{n}{n\,x + x^5 + 1}\,\mathrm{d}x,$$