Series 20

Jakub Woźnicki

02.06.2023

Exercises:

(A1) Calculate the integral

$$\int_0^1 \left(x \left\lfloor \frac{1}{x} \right\rfloor + x - 1 \right) \left\lfloor \frac{1}{x} \right\rfloor \mathrm{d}x.$$

(A2) Calculate the following limits

a)
$$\lim_{n\to+\infty} \int_0^\infty \frac{(\sin x + \cos x)^n}{1 + (\sin x + \cos x)^{2n}} e^{-2x} \,\mathrm{d}x,$$

b)
$$\lim_{n\to+\infty} \frac{1}{n} \int_1^\infty \frac{\mathrm{d}x}{x^2 \ln(1 + \frac{x}{n})},$$

c)
$$\lim_{n\to+\infty} \int_{-2\pi n}^{2\pi n} \left(1 + x^2 + \frac{x^4}{2!} + ... + \frac{x^{2n}}{n!}\right) \cos\left(\frac{x}{\sqrt{n}}\right) e^{-2\,x^2} \,\mathrm{d}x,$$

d)
$$\lim_{n\to+\infty} \int_0^\infty \frac{\ln(x^n + 2^n)}{n\,x^2} \,\mathrm{d}x.$$

(A3) Given a set A, calculate the limit.

a) $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. Calculate

$$\lim_{n\to+\infty} \int_A \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}} (x^n + y^n) \,\mathrm{d}\lambda_2(x, y).$$

b) $A = \{(x, y, z) \in \mathbb{R}^3 : 0 < x^2 + y^2 < 1,\ 0 < z < 2\}$. Calculate

$$\lim_{n\to+\infty} \int_A \frac{z}{n} \ln((x^2 + y^2)^n + (x^2 + y^2)^{-n}) \,\mathrm{d}\lambda_3(x, y, z).$$

c) $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1,\ x^2 + y^2 \geq z^2,\ z > 0\}$. Calculate

$$\lim_{n\to+\infty} n^2 \int_A \ln\left(1 + \frac{x^2 + y^2}{n}\right) \ln\left(1 + \frac{z}{n}\right) e^{x^2+y^2-z^2} \,\mathrm{d}\lambda_3(x, y, z).$$