Series 21

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Exercises:

(A1) Calculate $\det DF$ for the following substitutions

a) $F : \mathbb{R}_+ \times (-\pi, \pi) \times \mathbb{R} \to \mathbb{R}^3$, given by

$$F(r, \phi, z) = (r\cos\phi, r\sin\phi, z).$$

This is the so-called cylindrical substitution.

b) $F : \mathbb{R}_+ \times (-\pi, \pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}^3$, given by

$$F(r, \phi, \theta) = (r\cos\theta\cos\phi, r\cos\theta\sin\phi, r\sin\theta).$$

This is the so-called geographical spherical substitution.

c) $F : \mathbb{R}_+ \times (-\pi, \pi) \to \mathbb{R}^2$, given by

$$F(r, \phi) = (r\cos\phi, r\sin\phi).$$

This is the so-called polar substitution.

(A2) Calculate the measure of the set $C = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 < 1,\ y^2 + z^2 < 1\}$.

(A3) Calculate the integral

$$\int_{\{0<y<2x\}} (1 + x - y)e^{-x}\, \mathrm{d}\lambda_2(x, y).$$

(A4) Calculate the integral

$$\int_A \frac{1}{(x + y + 1)^2}\, \mathrm{d}\lambda_3(x, y, z),$$

where $A = \{(x, y, z) \in \mathbb{R}^3 : 0 < x < 1,\ 0 < x + y < 1,\ 0 < z < \frac{1}{x+y+1}\}$.

(A5) Calculate the integral

$$\int_{\{x>0, y>1, z>1\}} \frac{x}{1 + (xyz)^4}\, \mathrm{d}\lambda_3(x, y, z).$$