

Series 22

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Exercises:

(A1) Calculate the measure of the set $C = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 < 1, y^2 + z^2 < 1\}$.

(A2) Calculate the area of the region bounded by the curve $(x^2 + y^2)^{\frac{3}{2}} = y$.

(A3) Calculate the measure of the set $D = \{(x, y) \in \mathbb{R}^2 : (x^2 + y^2)^2 \leq 4(x^2 - y^2)\}$.

(A4) Calculate the integral

$$\int_{\{0 < y < 2x\}} (1 + x - y)e^{-x} d\lambda_2(x, y).$$

(A5) Calculate the integral

$$\int_{\{x > 0, y > 1, z > 1\}} \frac{x}{1 + (xyz)^4} d\lambda_3(x, y, z).$$

(A6) Calculate the integral

$$\int_{\{x^2 + y^2 \leq y, x \geq 0\}} \frac{x}{\sqrt{x^2 + y^2}} d\lambda_2(x, y).$$

(A7) Let $a < b$ be positive numbers. Calculate the measure of the set

$$E = \left\{ (x, y) \in \mathbb{R}^2 : 1 \leq xy \leq 2, a \leq \frac{y}{x^2} \leq b \right\}.$$

(A8) Calculate the integral

$$\int_A x^4 y d\lambda_2(x, y),$$

where $A = \{(x, y) \in \mathbb{R}^2 : \frac{1}{3}y^2 < x < \frac{1}{2}y^2, \frac{1}{x} < y < \frac{2}{x}\}$.

(A9) Calculate the integral

$$\int_A x e^{-y^2} d\lambda_2(x, y),$$

where $A = \{(x, y) \in \mathbb{R}^2 : |x + y| < 1, y > 0\}$.