Improved Approximation Ratio for Strategyproof Facility Location on a Cycle

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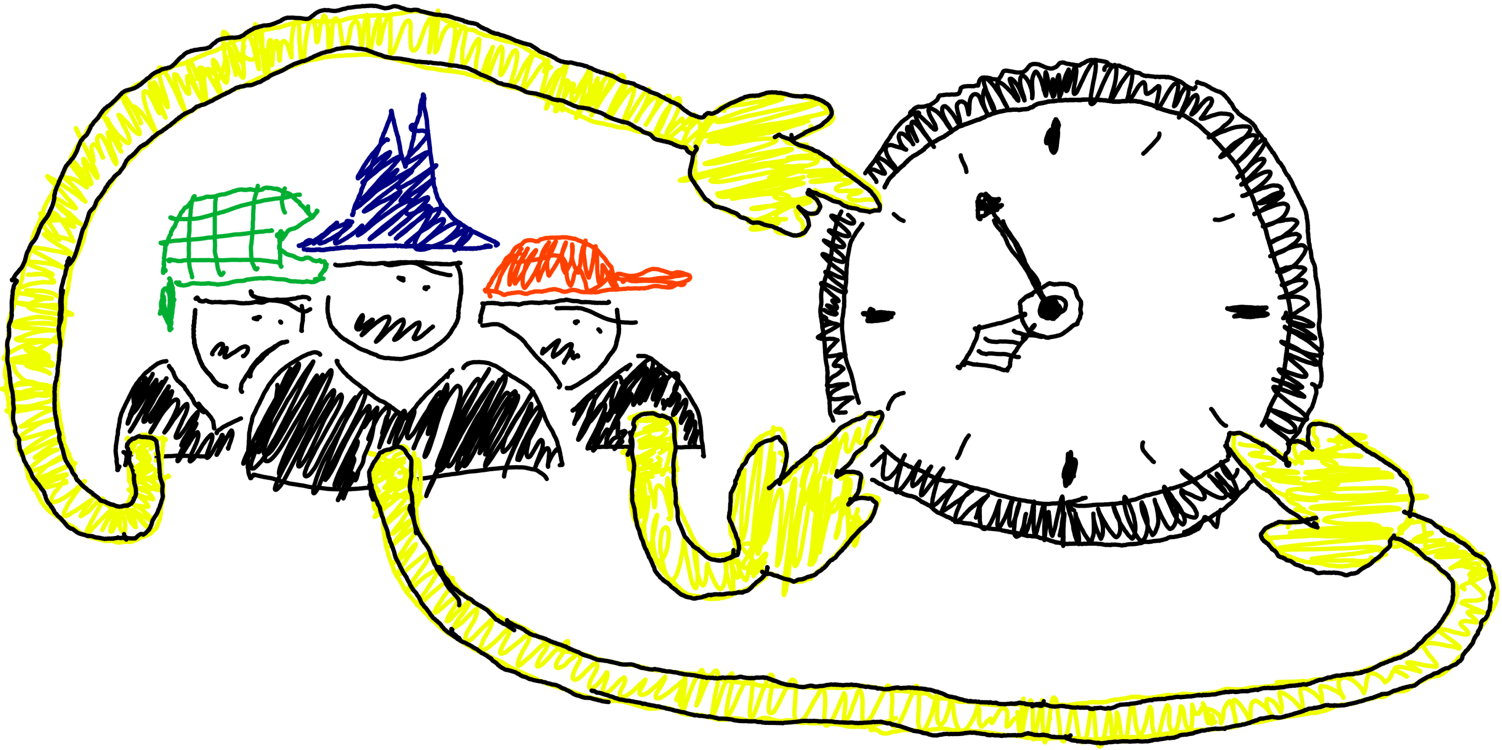
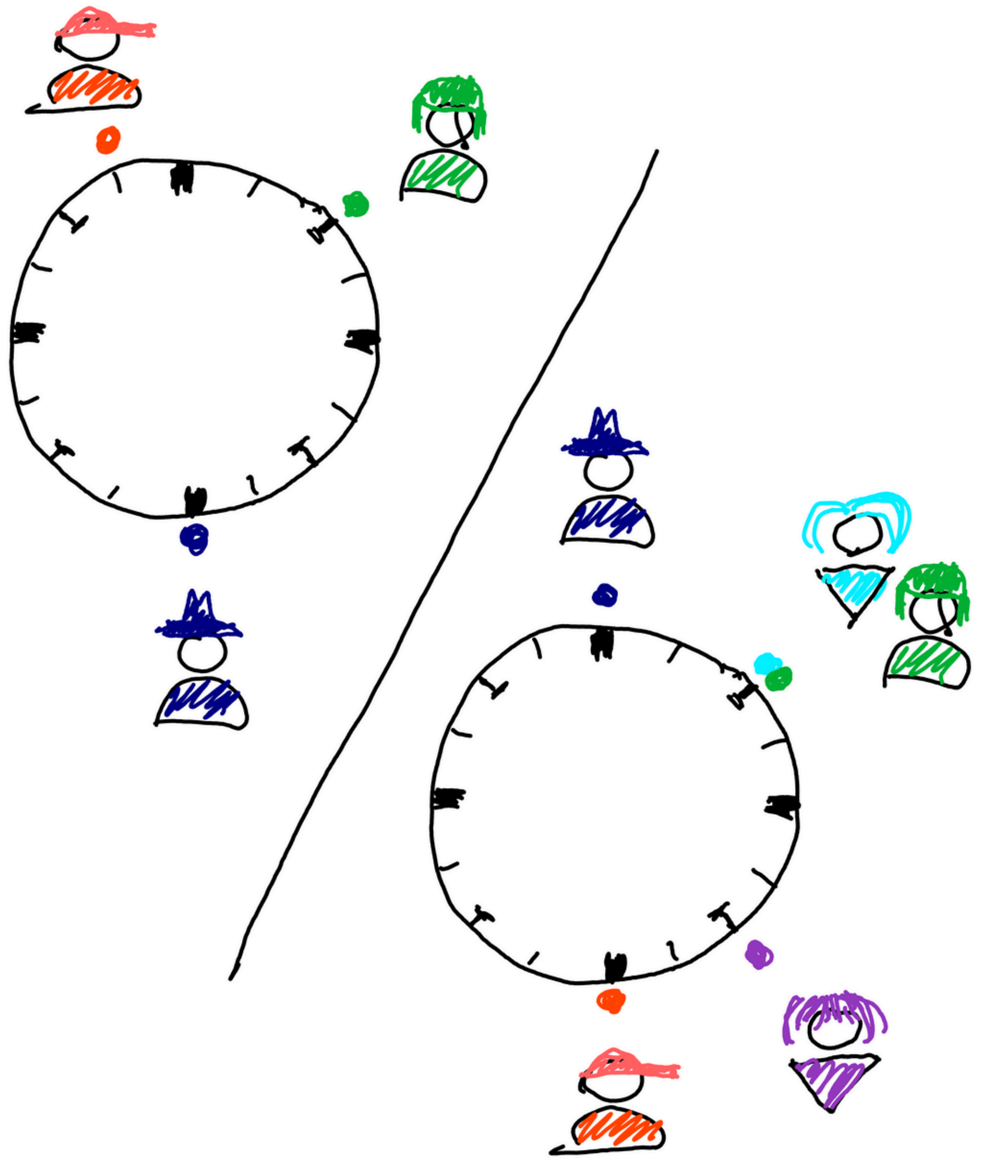


Motivating Examples

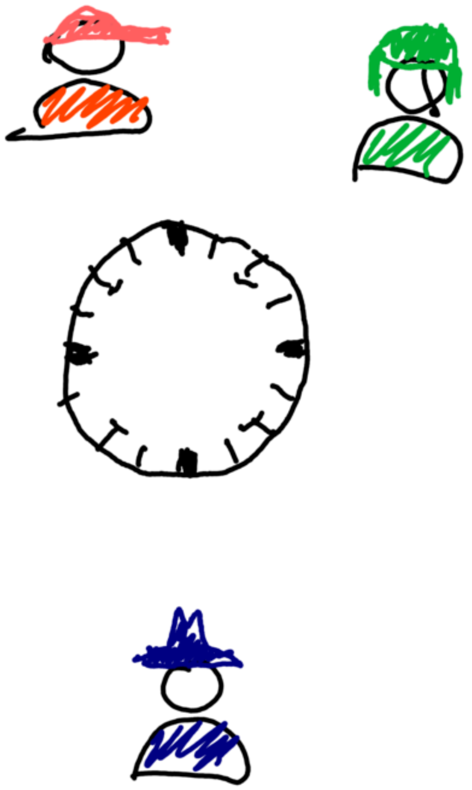
Selecting:

- **time of the day**
- political candidate
- location of a facility



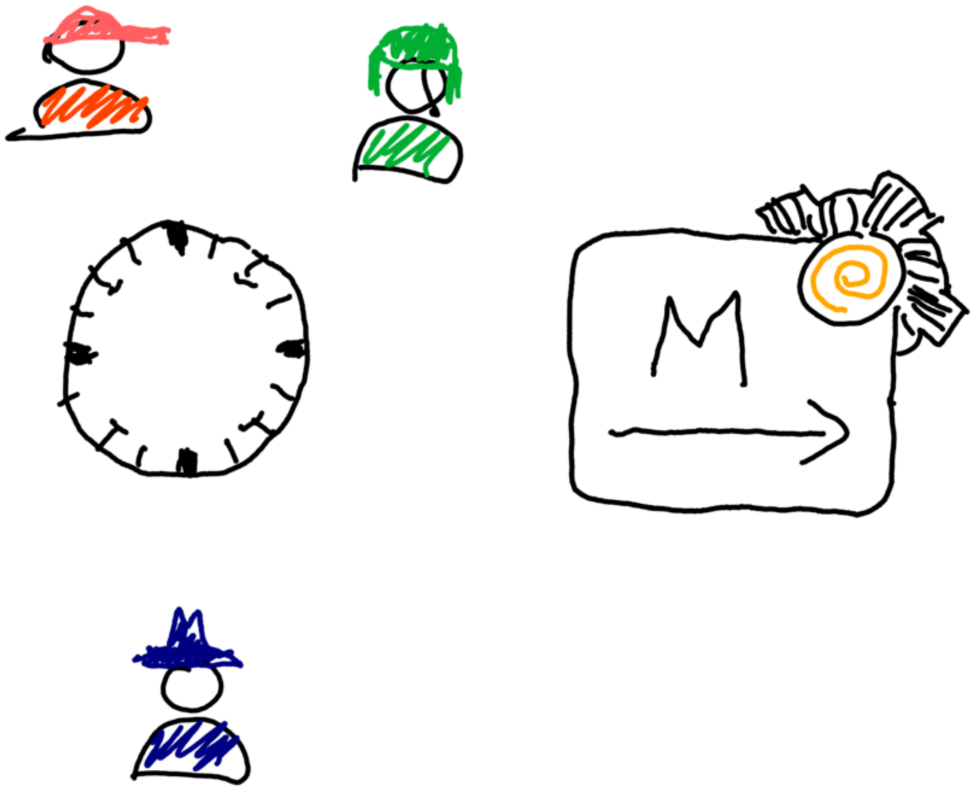
Model

Cyclic graph G. Set of agents $N = \{1, \ldots, n\}$, n odd.



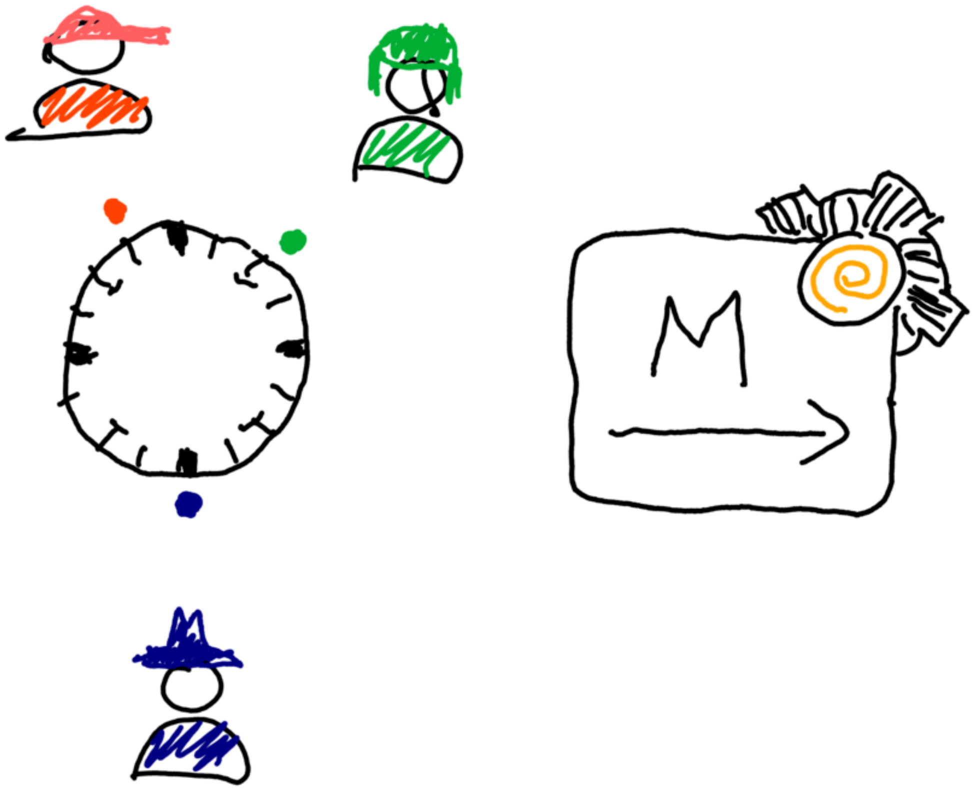
Model

Mechanism $M : G^N \to \Delta(G)$, maps votes of agents to a lottery over vertices.



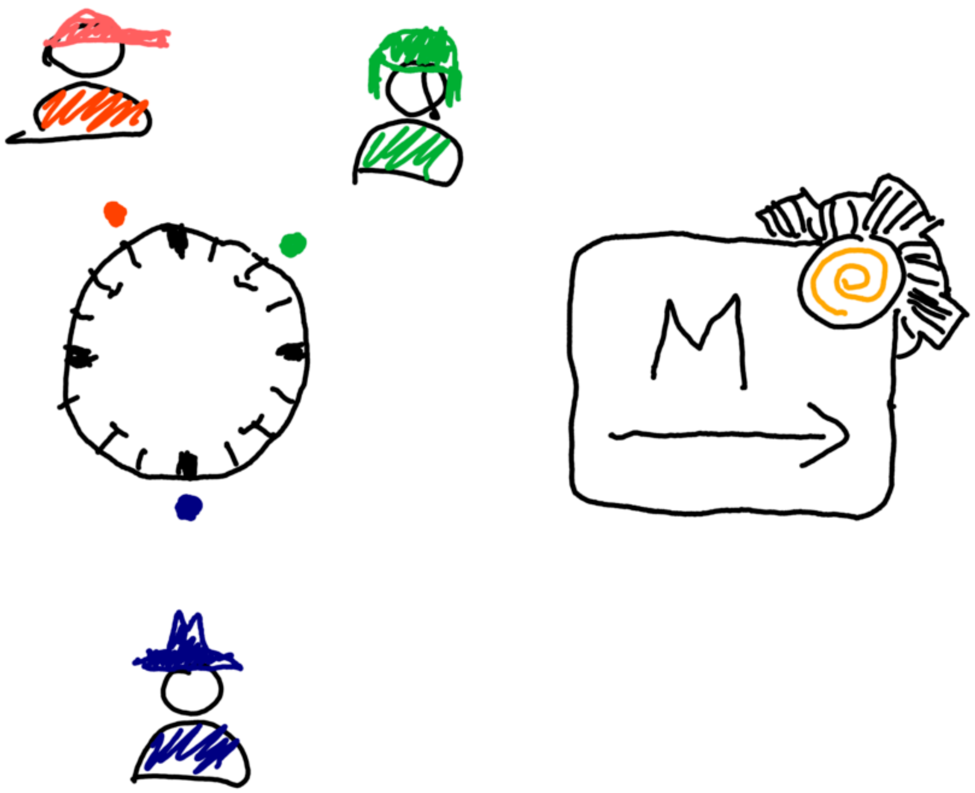
Model

Agents has preferences over vertices of graph G induced by their ideal vertices.



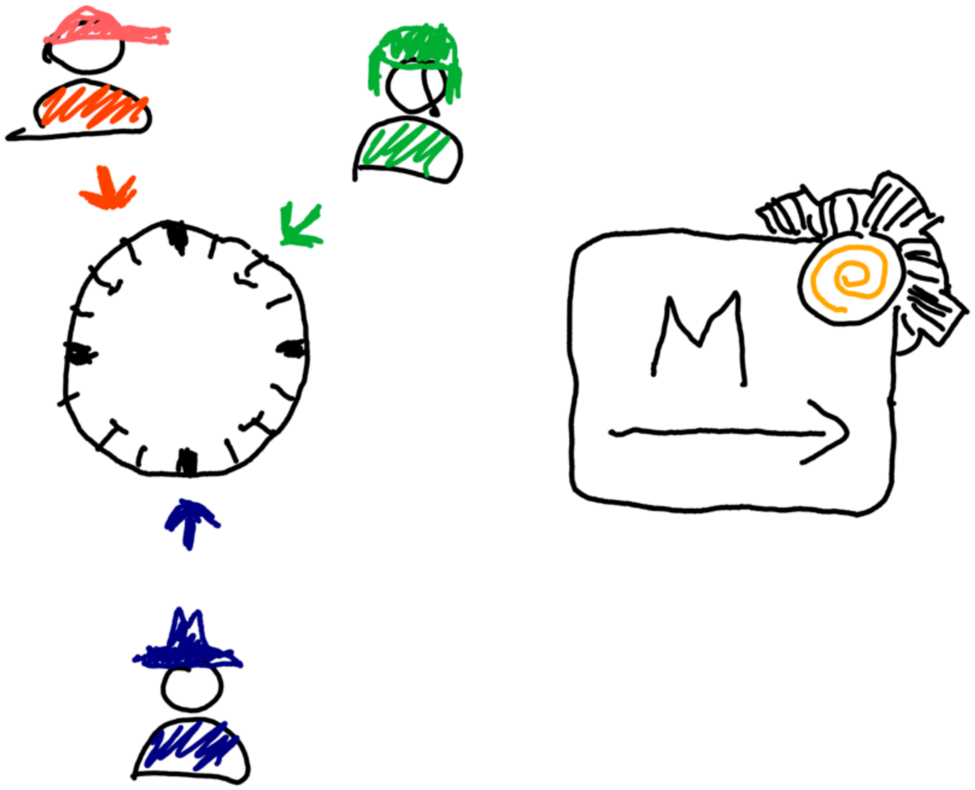
Model

Cost incurred by agent from selecting vertex is its distance from their ideal vertex.



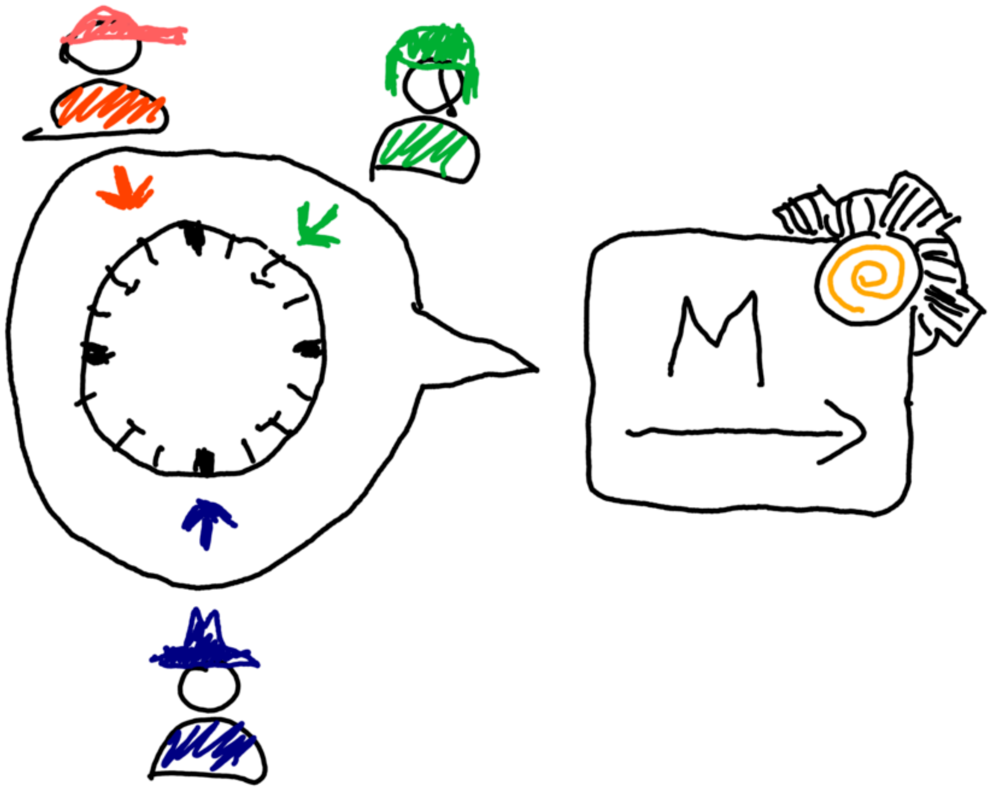
Model

Agents observe mechanism first, then submits their votes (any vertices of G).



Model

Mechanism selects a resulting lottery based (only) on the votes of agents.



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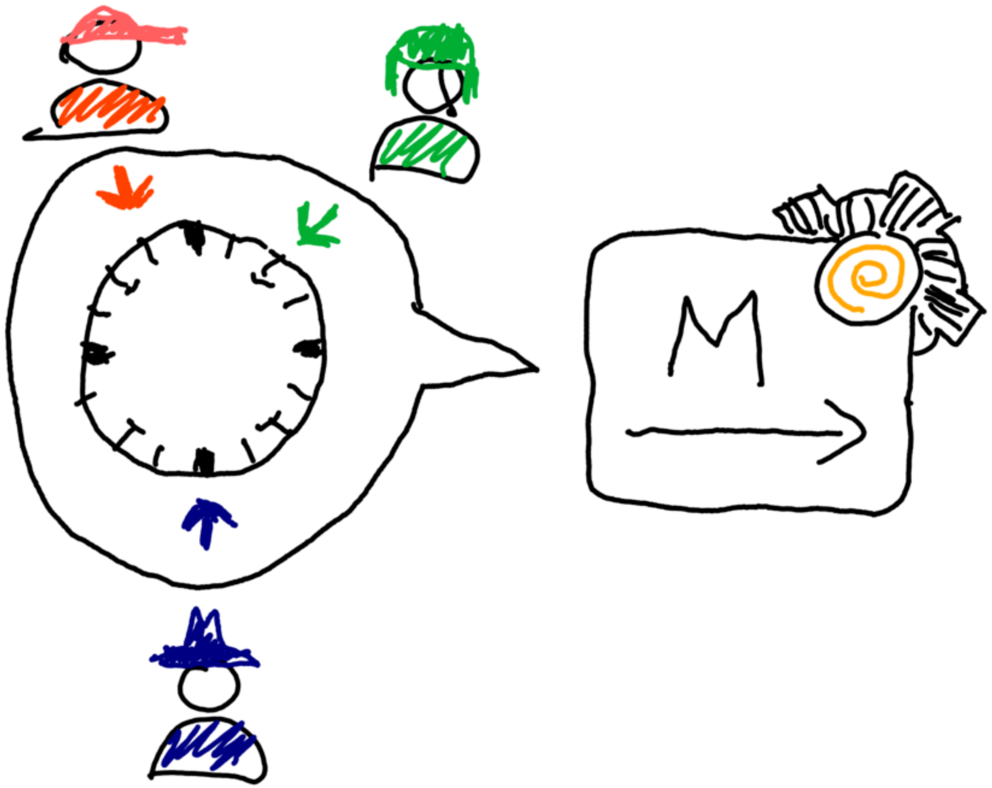
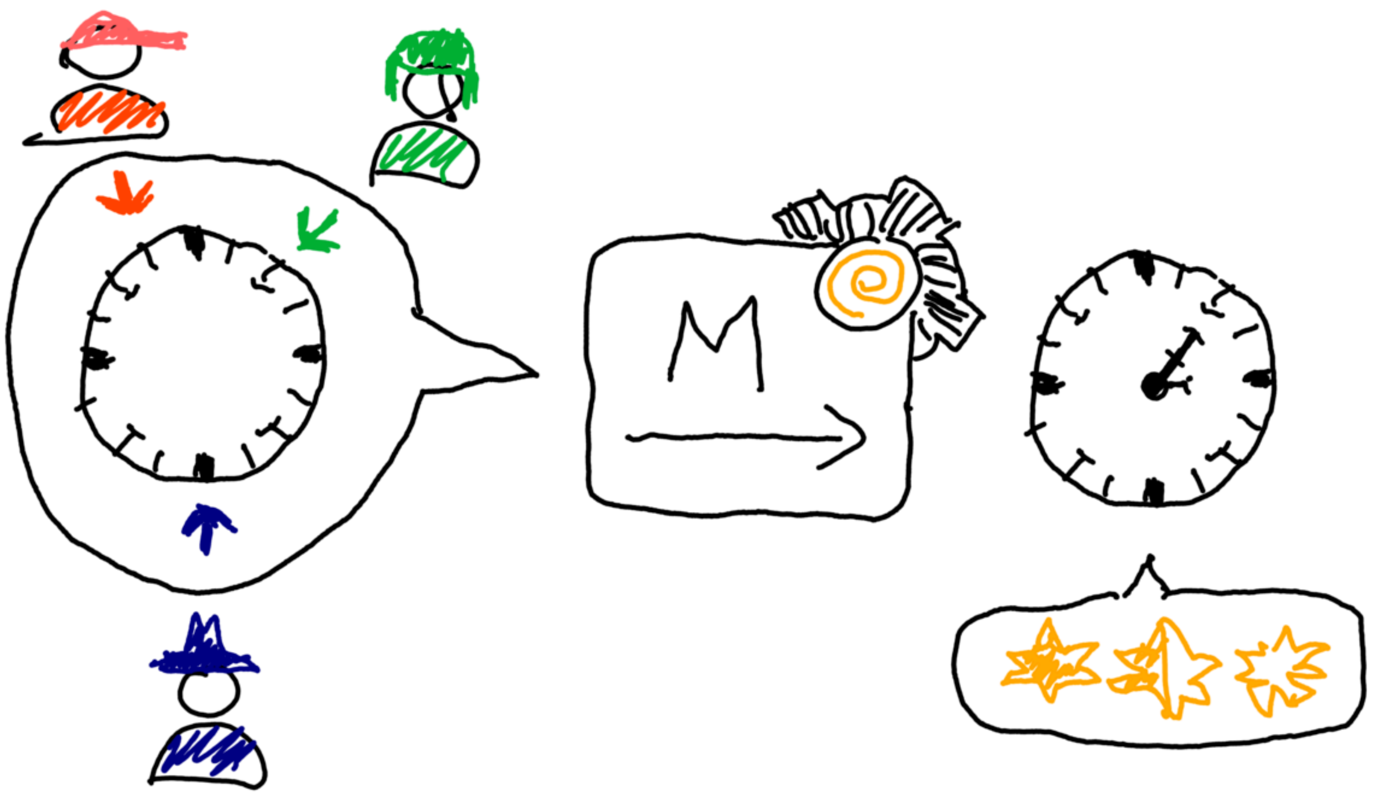


Model

Agents and social planner perceives mechanism based on the resulting lottery.



Model (costs)

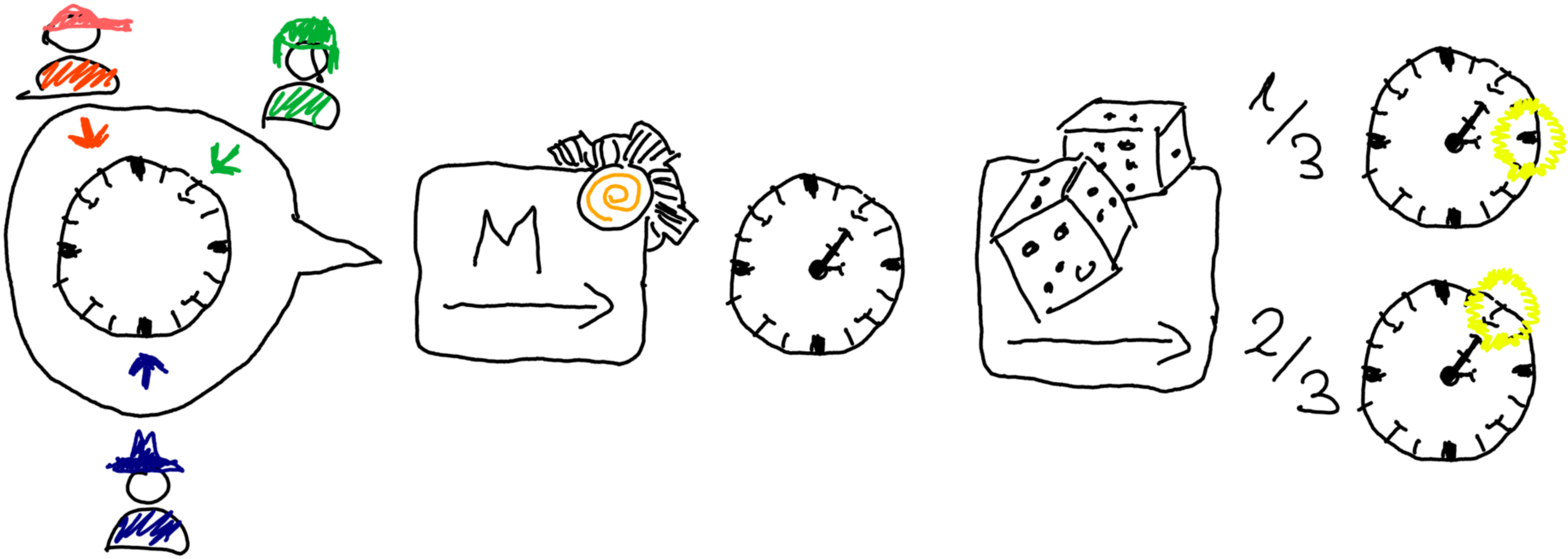
Let $L \in \Delta(G)$ be a lottery distribution over the vertices of G.

Cost of agent i from L: $c_i(L) = \mathbb{E}_{v \sim L}[d_i(v)]$

Social Cost of L: $sc(L) = \sum_i c_i(L)$

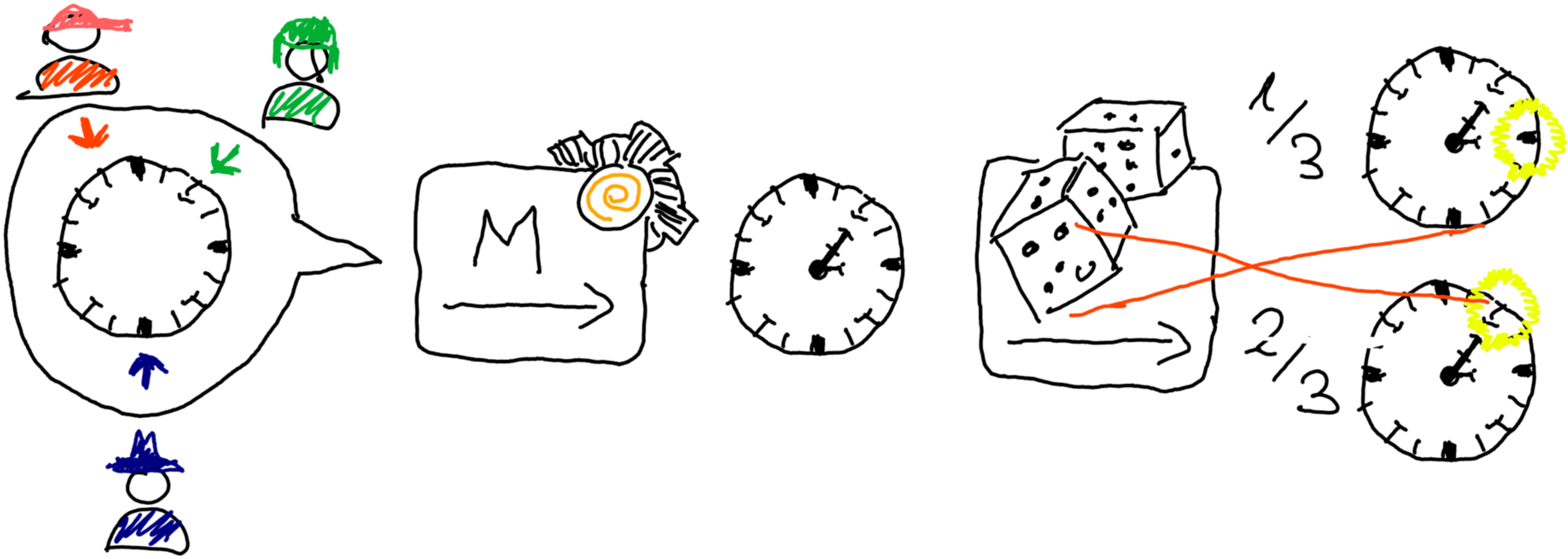
Model

Result of the selection is sampled from the lottery returned by the mechanism.



Model

Random sampling of the lottery is not interesting.



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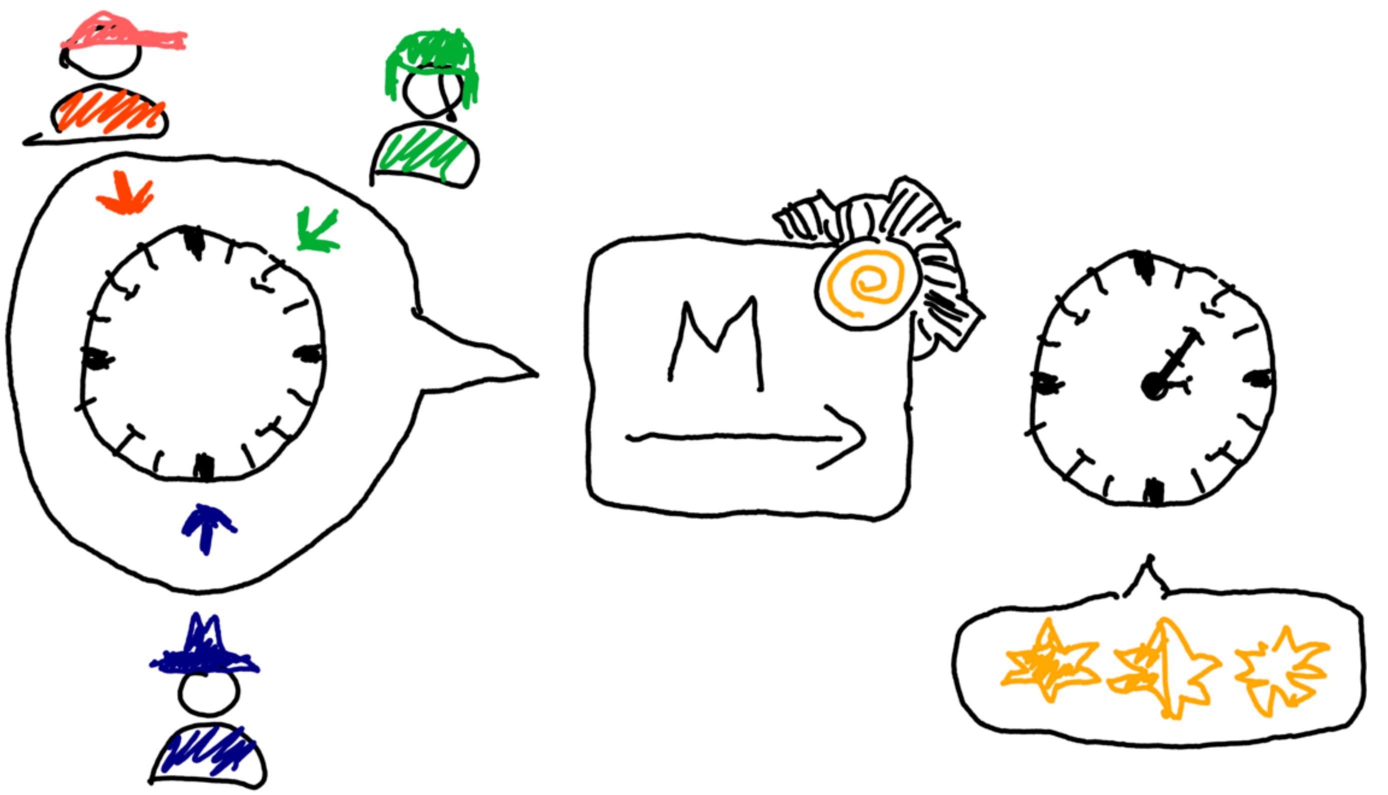
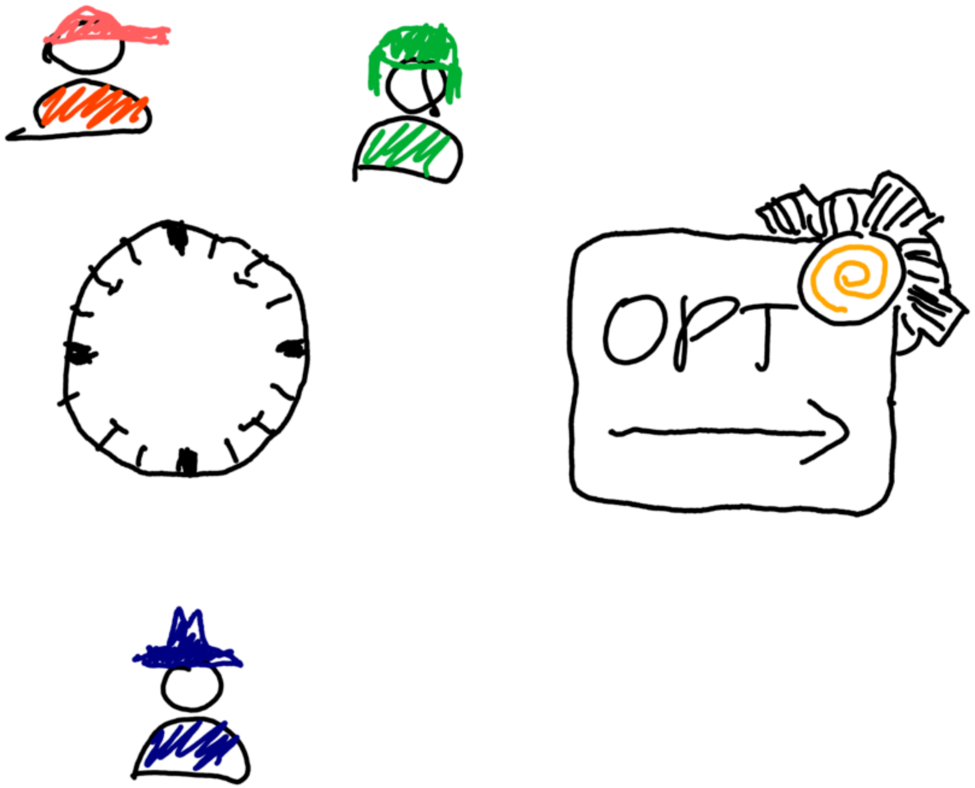


Opt Mechanism

Consider opt mechanism which returns some lottery over socially optimal vertices.



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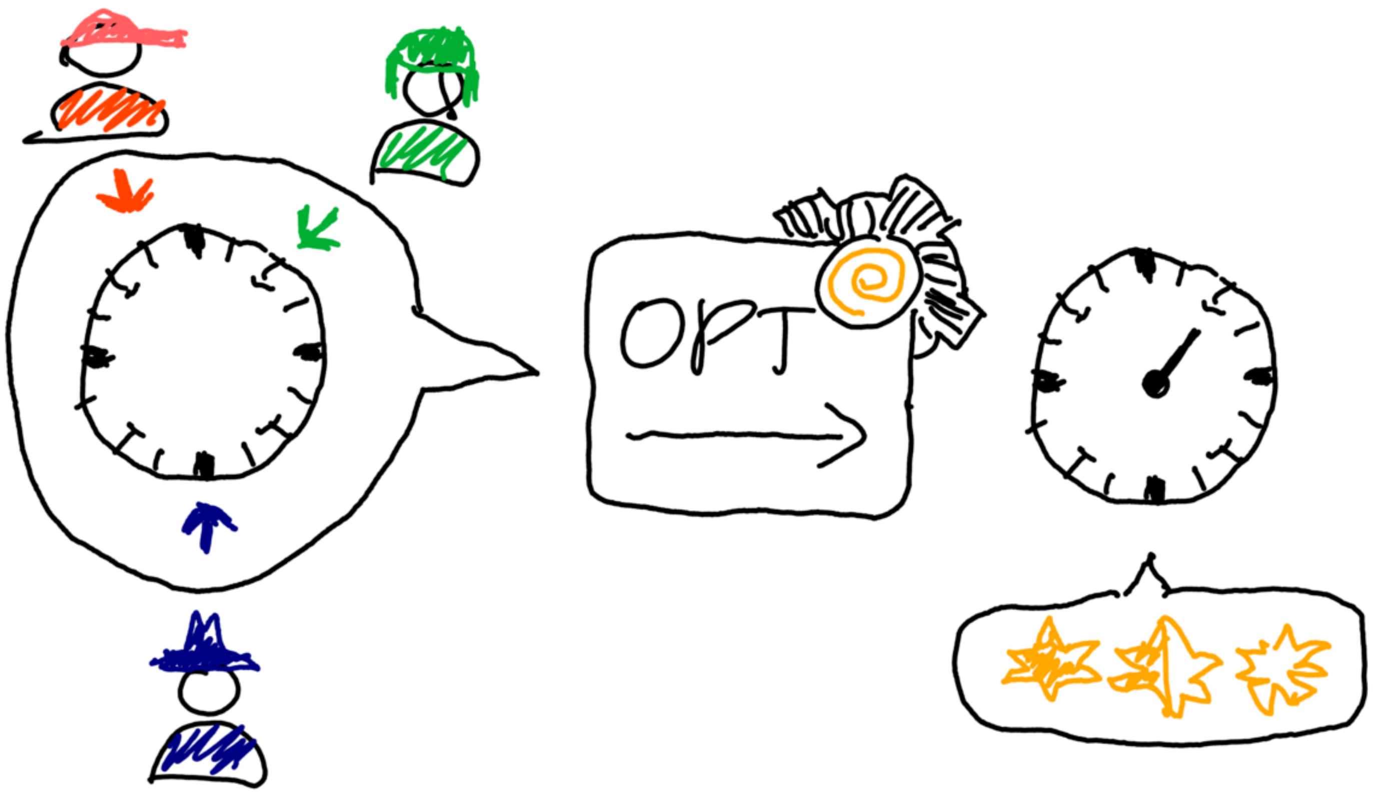
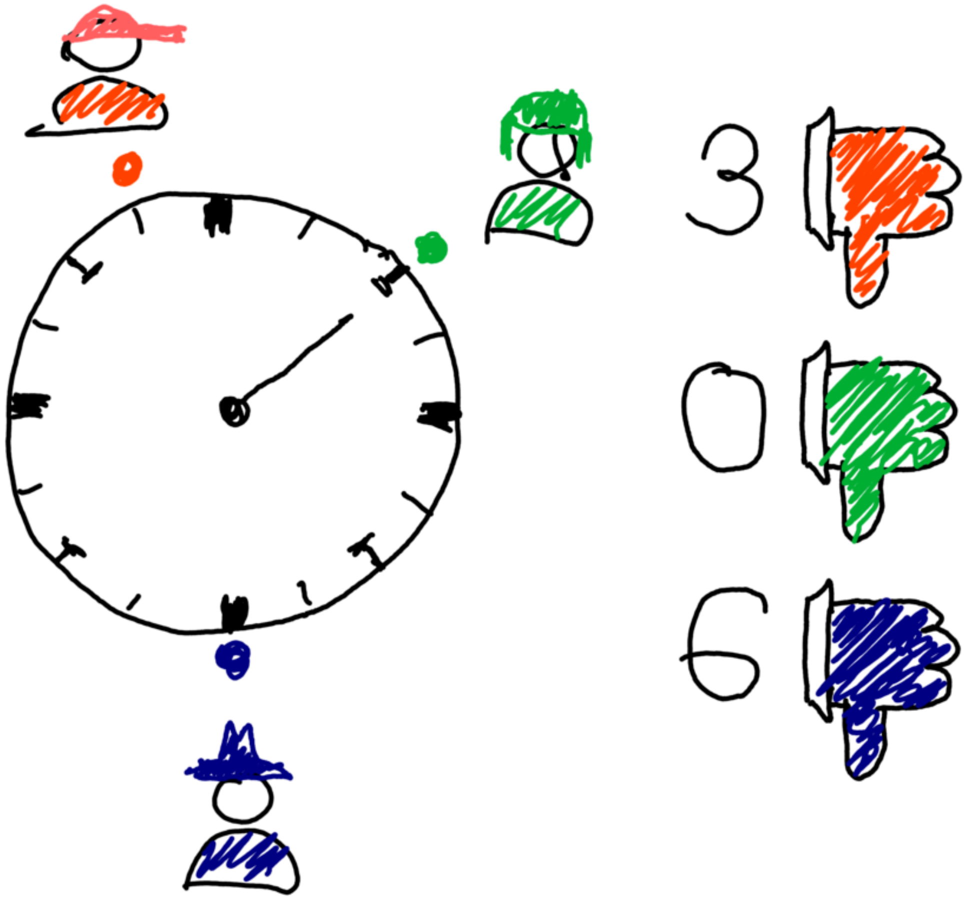
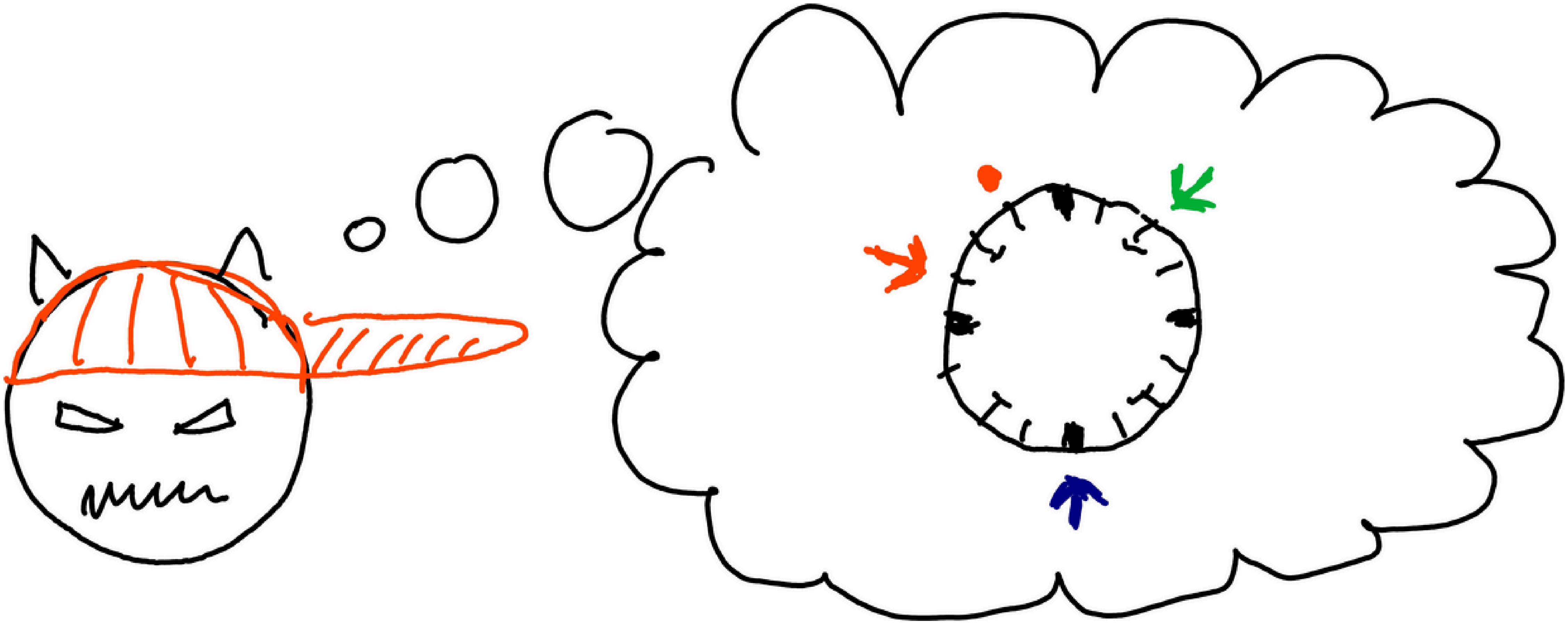


Agent costs

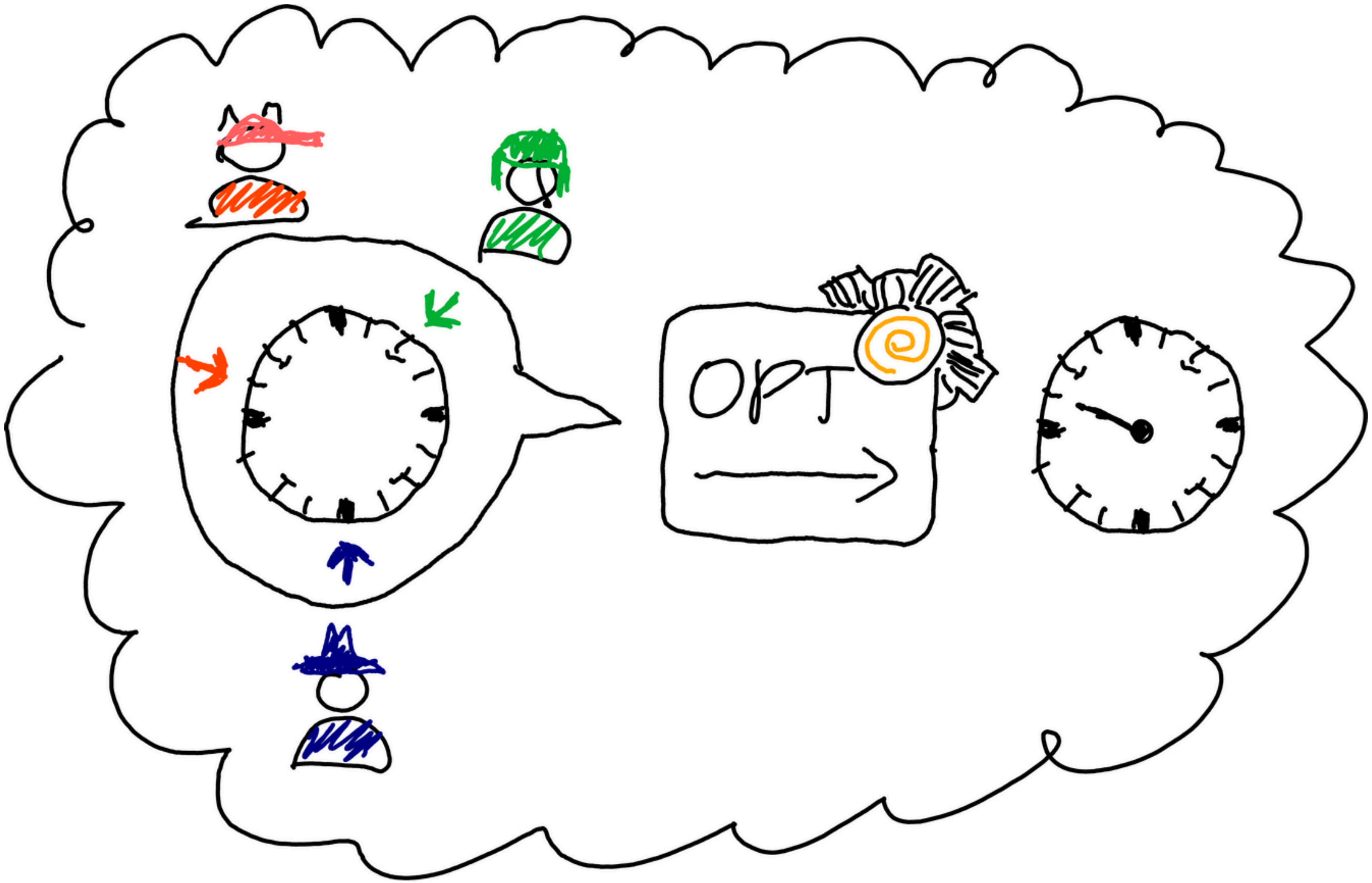


Deviations

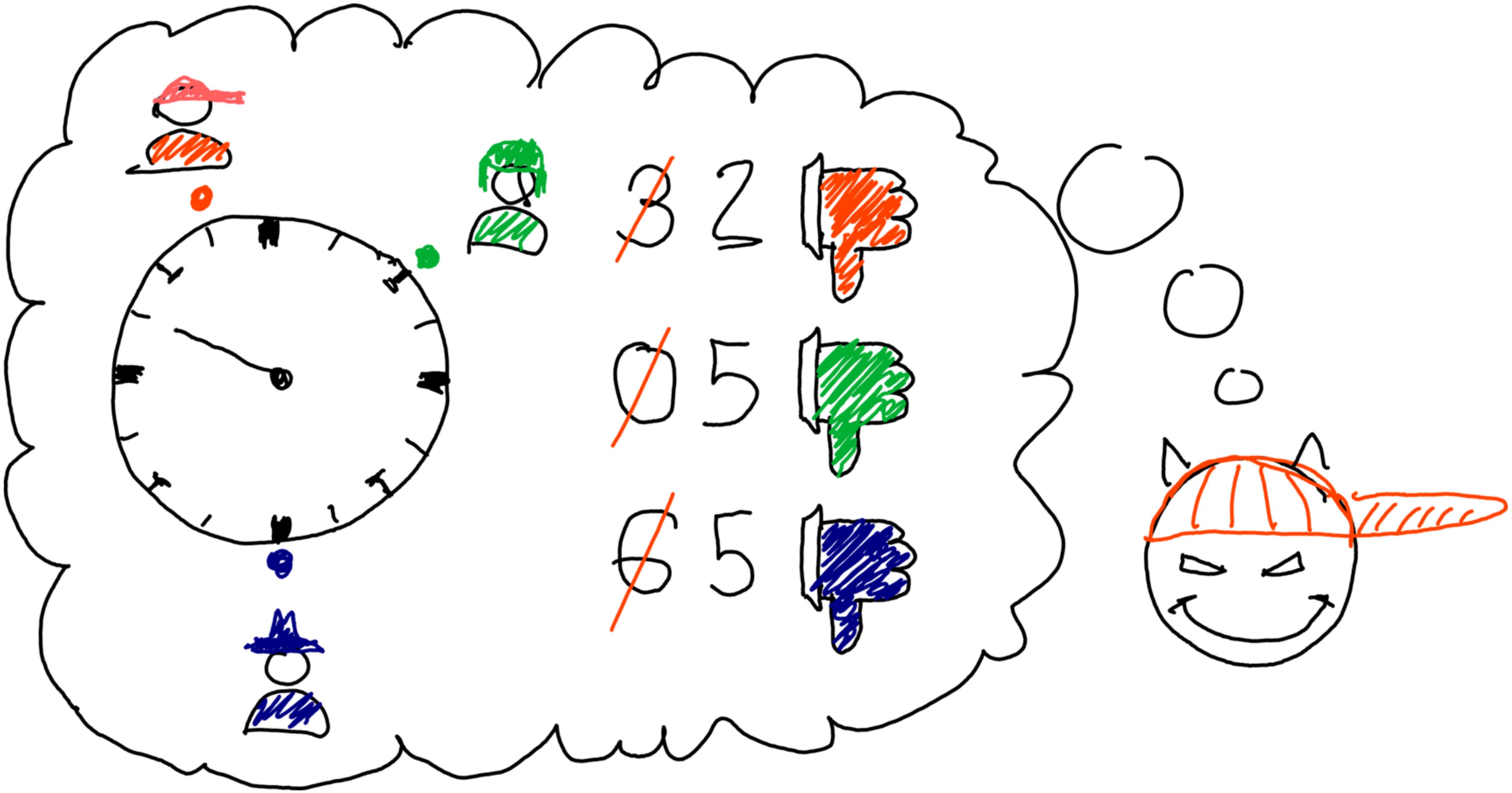
Any agent can misreport their vote to try to minimize their own cost.



Deviations



Deviations



Strategyproofness (in expectation)

Intuition: No one can benefit from misreporting their vote.

Definition: Mechanism M is strategyproof if for any state of the world $s \in G^N$, any agent $i \in N$ and alternative report v_i of agent i it holds that:

$$c_i(M(s)) \leq c_i(M(s_{-i}, v_i))$$

Assumption

Further we restrict ourselves to mechanisms which are strategyproof.

Implications: We can assume that agents report their votes truthfully.

Note: Mechanism returning optimal vertex may not be strategyproof on the cycle (depending on number of vertices).

Question: What is the best strategyproof mechanism?

Approximation Ratio

Intuition: Measure how good the mechanism is by comparing the social cost of its outcome to optimal outcome in the **worst case**.

Definition: Minimal constant ϕ such that for every state of the world s it holds that:

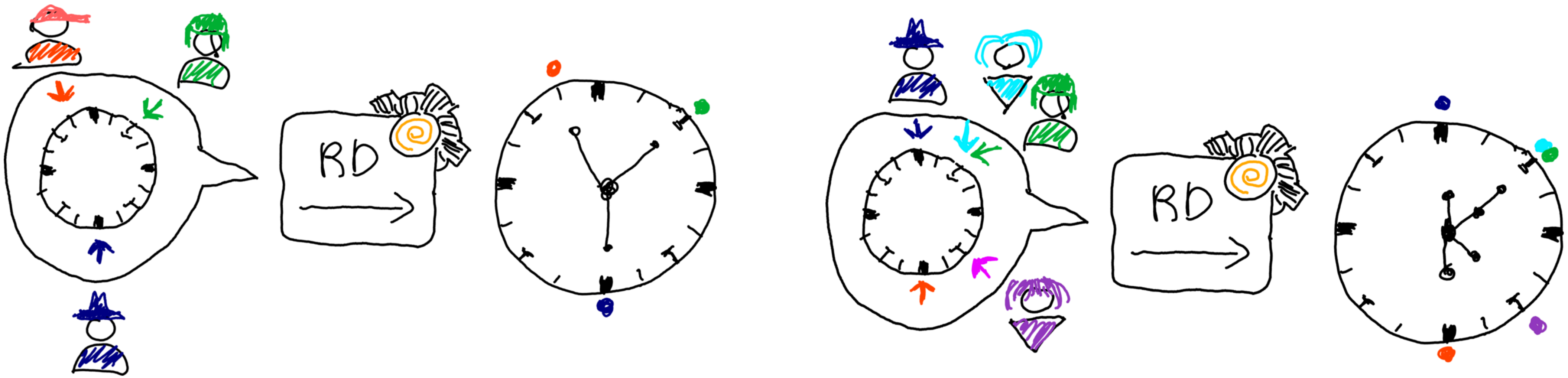
$$sc(M(s)) \leq \phi \cdot opt(s)$$

RD Mechanism

Working: Selects each agent's vote with the same probability (uniform lottery).

Properties: strategyproof, neutral, anonymous

Approximation Ratio bound: $2 - 2/n$

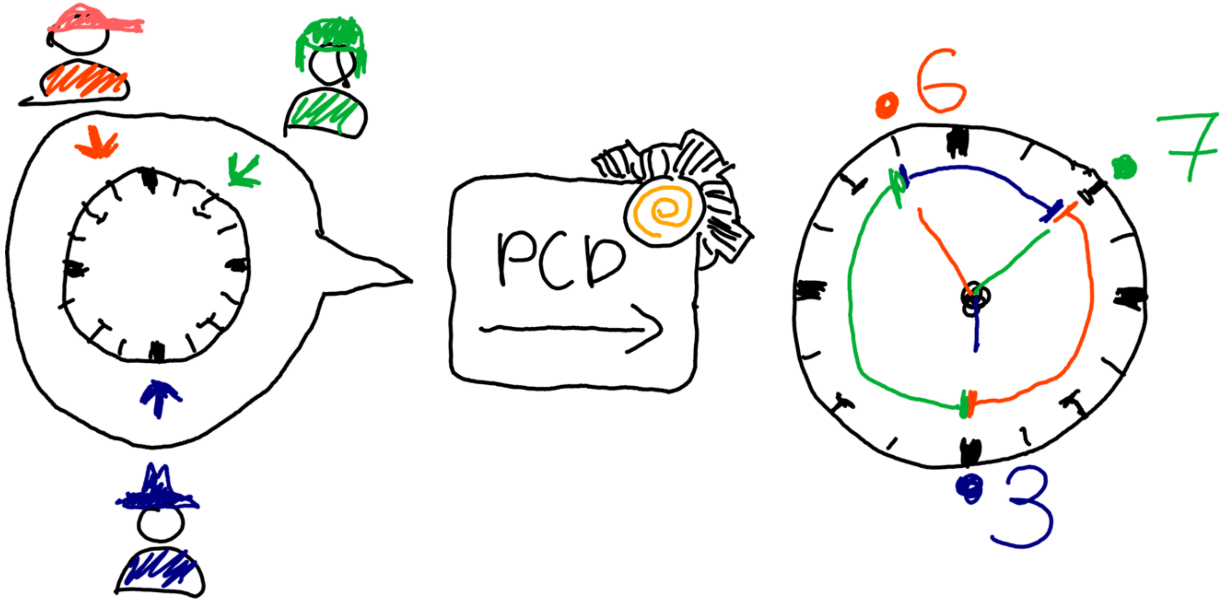


PCD Mechanism (Meir 2019)

Working: Selects each agent's vote with probability proportional to length of the opposing arc.

Properties: strategyproof, neutral, anonymous

Approximation Ratio bound: 2

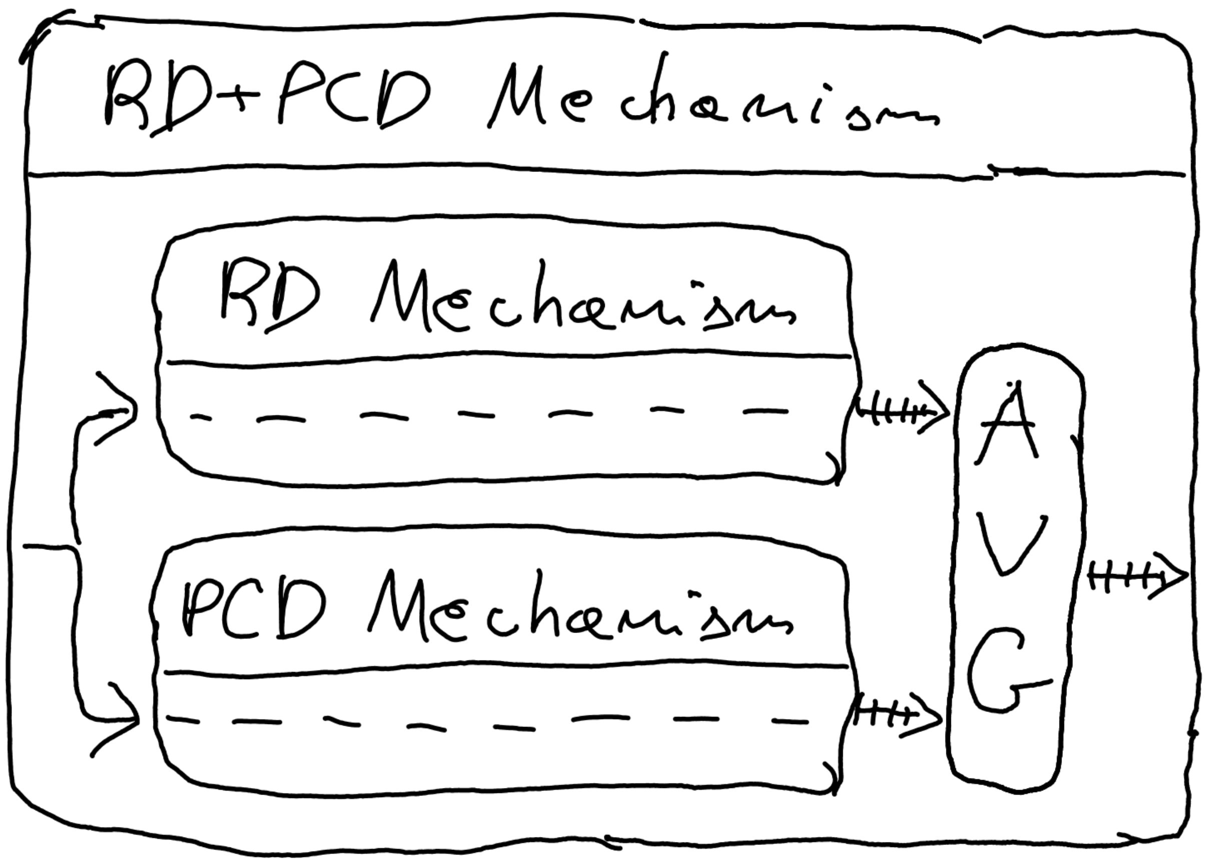


New stuff from now on

Mixed Mechanism RD+PCD

Definition: Consider a RD+PCD mechanism which for each state of the world returns average lottery of RD and PCD.

Preserves: strategyproofness, neutrality, anonymity (if both input mechanisms adhere to these properties)



Approximation ratio of RD+PCD

Observation: For every state s approximation ratio of RD+PCD is average of approximation ratios of RD and PCD.

Let s be the state of world for which RD performs poorly (poorer than PCD). Consider possible performance of different mechanisms for s.

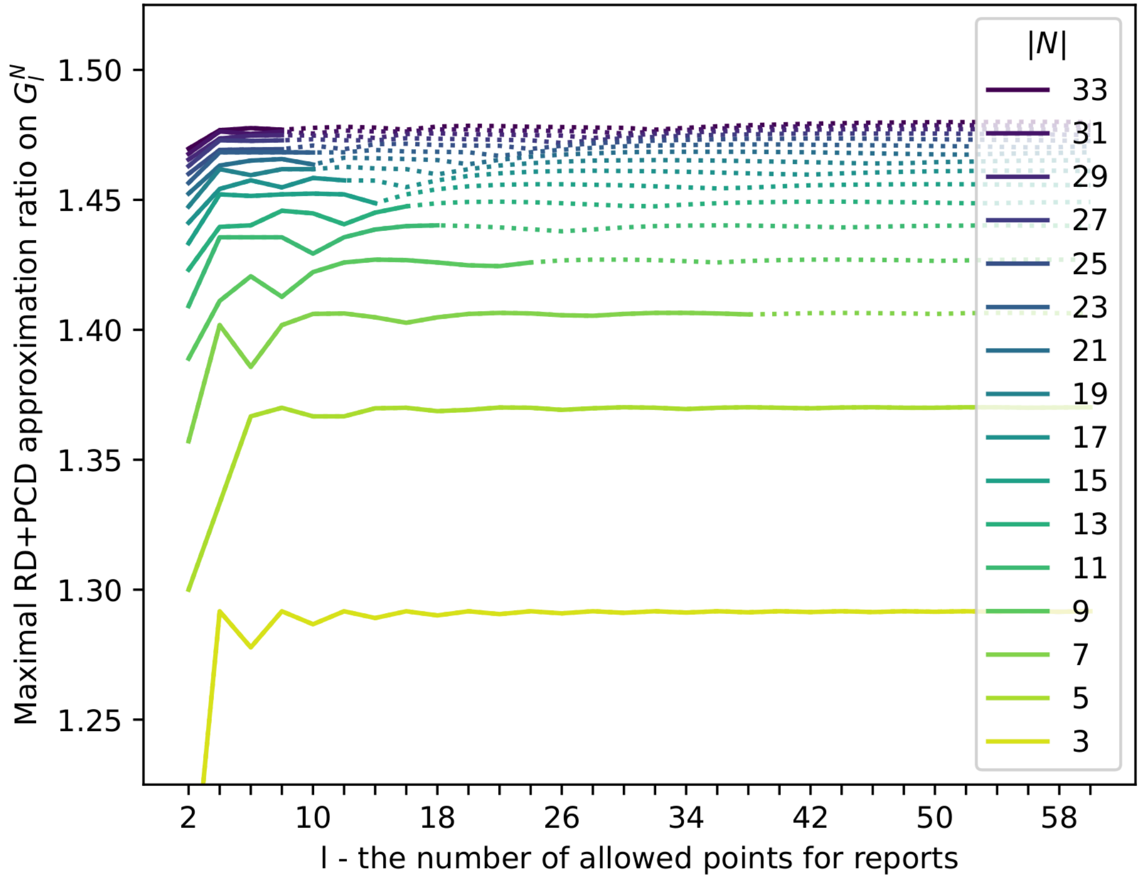
Performance (apx)	RD	PCD	RD+PCD
Pessimistic	$\lessapprox 2$	< 2	< 2
Optimistic	$\lessapprox 2$	< 1	< 1.5

This works for all states of the world and mechanism switched.

Approximation ratio of RD+PCD

Bounds for approximation ratio:

- trivial: 2,
- optimistic: 1.5,
- experimental: 1.5 (!),
- **proven:** 1.75.



Results

Theorem: For any set of n agents with an odd cardinality and a cyclic graph G, there exists a strategyproof mechanism for single facility location whose approximation ratio with respect to social cost is bounded from above by $\phi = 1.75$.

Results

	ϕ
State of the Art (RD mechanism)	$2 - 2/n$
Our Result	1.75
Our Experiments	1.5
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Lower Bound (Meir 2019)	1.045

Further Work

- Examination of experimental bound of 1.5 for approximation ratio of RD+PCD.
- Generalization to other classes of graphs.
- Research mixes of different mechanisms.

Proof technique (thanks)

- Fixing vertex generating optimal social cost (due to neutrality and anonymity).
- Cutting the cycle (estimating the distance on the cycle by the distance on a line segment).
- Identifying variables in which bound is monotonic.
- Direct estimating.